How Play in a Nature Explore Classroom Supports Preschool and Kindergarten-Age Children's Math Learning: A Single Case Study at an Early Education Program in Nebraska

Authors:
Dr. Dana L. Miller, Research Consultant
Doane College School of Graduate and Professional Studies

Kathy Tichota, Teacher/Co-Researcher
Dimensions Early Education Programs

Joyce White, Teacher/Co-Researcher
Dimensions Early Education Programs

May 2014

Acknowledgements

With great admiration, respect and special thanks to the 19 teacher/co-researchers who contributed documentation to this study:

Laurie Flynn
Heather Guess
Suzan Haley
Cindy Heinzman
Kristin Holmes
Barb Jensen
Donalyn Katt
Amanda Kelly
Chris Kiewra
Katie Logan
Sherry Miller
Holly Murdoch
Renee Pracheil
Natalie Ragland
Tina Reeble
Kristi Reitz
Kathy Tichota
Kris Van Laningham
Joyce White

This research would not exist without your commitment and expertise. We are grateful for your Nature Notes, sketches, and photographs and especially for the work you do each day, making a difference in children's lives.

Prologue

"Why do people like me abhor math? Because it makes little to no sense. Nothing about it is "intuitive," as my math teachers (all the way back to middle school through high school) said during my math classes…Case in point: fractions. To me, fractions were a mystery – why was 1/3 smaller than ½ when 3 was more than 2? Intuitive? Not to me, not back then!"

~ D.Y. Archie, June 11, 2013

"I hate math in general. I've never been good at it and I have no plans to work in a job that requires extensive math skills. I don't enjoy any type of math at all."

~ Blake, April 21, 2014

"I learned a great deal from high school math: Stay away from it! I did not want to repeat the failure of geometry class. It worked. I earned four academic degrees – none in math, thank God!"

~ G DeMarse, August 5, 2013

"I hate math because I don't understand it, and it frustrates me. Admitting that I don't understand it is silly. I am an adult college student and I am at home in my bed studying for a math test now, and I hate it because I still don't understand it, even after studying all day…It is frustrating and confusing – and that, my dears, is why I hate math."

~ Tamara, September 27, 2013

"I am 14 years old and I am good at math. I never really liked math though. I could never relate to it. There aren't many reasons we need it in real life anymore, because we have ways to solve problems online these days."

~ Cheese pop, October 20, 2013

Responses to Annie Murphy Paul's "Brilliant Blog": Why do so many of us hate math?

Introduction

Why is it so important to help young children have positive experiences developing mathematical knowledge? The candid responses to Annie Paul's blog provide just a sliver of insight into a significant problem that plagues our nation. The statistics on math proficiency in the United States are bleak. According to the Program for International Student Assessment (PISA) U.S. students in the class of 2011 ranked below students in 31 other countries in math proficiency (the U.S. ranked 32nd out of 65 countries). In fact, less than one-third of U.S. students performed at a proficient level. Only six states had a proficiency level of 40% or above (Kansas, New Jersey, North Dakota, Vermont, Minnesota and Massachusetts). The state with the highest proficiency level was Massachusetts (50.7%) and the state with the lowest proficiency level was Mississippi (13.6%), with the District of Columbia even lower (8.0%). The state of Nebraska ranked 23rd, with a proficiency rate of 34.6% (Peterson, Woessmann, Hanushek, & Lastra-Anadon, 2011). Low levels of math achievement have been observed as early as first grade and kindergarten, with serious individual differences existing in acquiring basic mathematics knowledge as early as three years of age. Many young children have not had high quality opportunities to develop mathematics knowledge, and learning difficulties occur when children do not get to develop that knowledge through everyday, meaningful activities (Clements, Baroody, & Sarama, 2013).

One deterrent to developing math proficiency is math anxiety (feelings of tension, worry, fear and pressure that can significantly impede performance). Math anxiety is a growing concern because it affects how children view their abilities and learn mathematical applications. While preschool and kindergarten-age children are likely too young to experience math anxiety, research has suggested that "levels of mathematics anxiety increase from fifth grade throughout middle school, reaching peak levels in ninth and tenth grade" (Hembree, as cited in VuKovic, Keiffer, Bailey & Harari, 2012, p. 2). Much of the research on math anxiety has focused on its negative impact on students from second grade through college (Baloglu & Kocak, 2006; Birgin, Baloglu, Catlioglu & Gurbuz, 2010; Capraro, Capraro, & Hensen, 2001; Newstead, 1998; Suinn, Taylor & Edwards, 1988; Vakuvic, Kieffer, Bailey & Harari, 2012). Vukovic et. al. (2012) suggested that "math anxiety has consistently been found to have an inverse relation with mathematical performance…(and the) negative impacts of mathematics anxiety have far-reaching consequences: compared to their less anxious peers, mathematically anxious students enjoy mathematics less, have lower perceptions of their mathematical abilities and do not see the value of mathematics in every day life" (p. 1). Those who suffer from math anxiety are less likely to pursue math-related majors in college and employment in fields that require math proficiency. This is a serious concern. According to the Organization for Economic Cooperation and Development, "the U.S. ranks 23rd among developed nations in the number of science and engineering graduates. Meanwhile, job growth in STEM (science, technology, engineering and math) fields is outpacing non-STEM-related job growth by 300 percent (U.S. Department of Commerce, Economics and Statistics Administration)" (as cited in the Raytheon study, 2012, p. 3). Math anxiety and deficiencies in basic math skills are problematic because mathematical literacy will be one of the currencies of the future that can be exchanged for greater success in an increasingly global society.

How do we help young children develop early math skills that will provide them with a solid foundation, nurture a continued love of learning, and translate to later success in constructing and applying math knowledge? Several authors have posited that the best way to help develop this early foundation is through age-appropriate learning, which for young children means learning through play. Researchers have examined the cognitive, social, intrapersonal and physiological benefits of play and support the belief that young children learn best through play (e.g., Brown & Vaughn, 2009; Gellen, 2007; Ginsburg, 2007; Hirsh-Pasek & Golinkoff, 2003; Hirsh-Pasek, Golinkoff, Berk & Singer, 2009; Hohmann & Weikart, 1995; Linn, 2008; Paley, 2004). Play is a "fundamental component to the development of a healthy childhood" (Linn, 2008, p. 19) and is important to "every single developmental domain" (Hirsh-Pasek, et. al., 2009, p. xi). During the early childhood years play is "the dominant and directing mode of learning" (Elkind, 2007, p. 7), and "seems to be the driving force helping to sculpt how the brain continues to grow and develop" (Brown & Vaughn, 2009, p. 42). Hirsh-Pasek and Golinkoff (2003) suggested that "play is to early childhood what gas is to a car…It is the very fuel of every intellectual activity that our children engage in" (p. 214), and concluded that based on research examining the benefits of play "a new equation is in order: PLAY = LEARNING" (p. 208).

Authors describe play as a viable way to develop math knowledge. Elkind (1999) described young children as "active learners who learn math concepts by pursuing their own interests" (p. 2). By nature children are curious about the world and about math but their learning is not regimented – they have their own curriculum goals. Richardson (undated) proposed that "mathematical understanding should be developed through stories, songs, games and imaginative play" (p. 6). A report on the progress of educational reform (Clements, Education Commission of the States, 2013) suggested that when young children are given opportunities to learn, they "possess an informal knowledge of mathematics that is amazingly broad, complex and sophisticated" (p. 2). The report identifies children's play as the primary vehicle for that knowledge acquisition:

> When children 'play,' they are often doing much more than that. Preschoolers can learn to invent solutions to solve simple arithmetic problems, and almost all of them engage in substantial amounts of pre-mathematical activity in their free play…Through higher-level play, children explore patterns, shapes, and spatial relations; compare magnitudes; and count objects…These explorations through play are pre-mathematical" (p. 2).

Authors have also advocated for the need to get children outdoors because nature is a powerful context for learning. While many educators do not equate the outdoors with traditional academic learning, Broda (2007) suggested that "outdoor education is not a subject area, rather, it is an instructional tool that can be used to enhance instruction in a variety of disciplines" (p. 11). Keeler (2008) described the natural world as "a great friend and teacher to young children" because it "offers infinite opportunities for wonder and learning, with surprises around every corner" (p. 55). Gardner (1993) viewed the outdoors as "the most powerful audio-visual tool around" (p. 24).

There are a host of benefits associated with having positive experiences in nature and they are directly related to learning. Some of the benefits researchers have identified include:

• an increased sense of wonder and imagination (Cobb, 1977; Wilson, 1997),

• improved ability to focus and concentrate (including children with ADD) (Taylor, Kuo & Sullivan, 2001),

• increased powers of observation and creativity (Crain, 2001),

• improved awareness, reasoning and observation skills (Pyle, 2002),

• increased language and collaboration skills (Moore & Wong, 1997),

• increased concern for the environment (Palmer, 1993),

• increased development of all senses (Louv, 2005),

• improved intrapersonal skills such as self-confidence and self-efficacy (Dillon, Morris, O'Donnell, Reid, Rickinson & Scott, 2005),

• reduced stress and increased ability to deal with adversity (Wells & Evans, 2003), and

• increased motivation for lifelong learning (Wilson, 1997).

In the last 25 years, researchers have identified the outdoor environment as a rich context for mathematical learning (as well as learning in other academic domains) (e.g., Basile, 1999; Bezanson & Killion, 2001; Pappas, 1989; Richardson, undated; Sobel, 1998; Waliczek, Logan, Zajicek, 2003). The outdoors promotes authentic, hands-on, active learning experiences that encourage higher levels of critical thinking and creativity. Waliczek, Logan and Zajicek (2003) examined the impact of a structured outdoor environmental program on elementary students' creativity and critical thinking, and on their attitudes toward math and science. The researchers used Bloom's Taxonomy as a theoretical framework for their study, and concluded that children not only learned math and science at the lower levels of Bloom's Taxonomy but also demonstrated thinking at higher levels, including application, synthesis and evaluation. Their themes supported the idea of positive learning in the outdoor classroom, and their participants used words such as "enthusiasm," "fun," "exciting," "new," "interesting," and "enjoyable" to describe their learning experiences outdoors. The authors' main theme became "experiential learning" that participants described as "hands-on," "practical," and "real world/real life" (p. 687).

Basile (1999) described the outdoors as an authentic context for math learning, including the value of natural materials in helping children construct math knowledge:

> One context that has proven beneficial for encouraging the transfer of mathematical skills and processes, especially in young children, is the outdoor environment. Young children

love to be outdoors and explore the natural world. Traditionally exploring nature has been viewed as a science activity. However, mathematics abound in nature and many things found there can be used to teach mathematics as well as science: leaves can be classified and sorted, the coloration of insects can be used to study patterns, trees can be measured and drawn, and the petals of a flower can be counted and compared with those of other flowers. In the classroom, children use manipulatives to build an understanding of mathematical constructs. In an outdoor setting, children use items found in nature to build an understanding of both mathematics and science…the natural world can be used as an essential context for (mathematical) instruction and learning (p. 157).

The literature we reviewed for this paper suggests that:

- The U.S. has a long ways to go to develop a future generation of students and employees who are proficient in math;
- A very real and significant obstacle that needs to be addressed to begin to increase math proficiency levels is the anxiety associated with math, from mild distaste of the subject to intense aversion (and associated feelings of fear, frustration and failure);
- Young children need to develop a solid foundation for learning early, and need to learn in ways that are meaningful to them. Children learn through play – it is essential to the development of healthy brains.
- Spending time outdoors (including in unstructured, self-directed play) benefits children intellectually, physically, socially, and interpersonally, because nature provides a powerful context for learning; and
- While most math instruction occurs in the traditional classroom, increasingly authors have made the case for moving math outdoors, since nature and the uniqueness and

abundance of natural materials supply rich opportunities for learning.

While there is significant research on these general topics, most studies on developing math skills have focused on children in elementary and secondary schools, and instruction that occurs in traditional indoor classrooms. A number of early childhood publications have provided anecdotal articles that address the importance of helping preschool-age children develop foundational pre-math skills but few provide research-based evidence that describes how that learning happens for young children. Even fewer studies have explored how young children develop math skills and construct math knowledge in outdoor settings. Only a handful of studies, conducted by teacher/co-researchers and consultants associated with Dimensions Educational Research Foundation have discussed the math skills preschool-age children developed as they were engaged in authentic play in Nature Explore Classrooms in Nebraska, Minnesota and California (Bohling, Saarela, Miller, 2010; Bohling, Saarela, & Miller, 2013; Miller, 2007; Miller, Tichota, & White, 2013; Veselack, Cain-Chang, & Miller, 2010). Those studies focused on learning in all domains, and not exclusively on math. This research is the first to focus exclusively on math learning for young children in a Nature Explore Classroom. Our goal was to explore how preschool and kindergarten-age children developed math skills as they worked and played in a Nature Explore Classroom.

The findings of this study will be of interest to several audiences. It will help early childhood educators better understand the math learning that takes place outdoors when children are allowed to follow their own interests and investigate the many ways math occurs naturally in nature. Because of this understanding, teachers may be able to enrich, enhance, and expand children's math experiences by providing interesting materials that support math learning, asking open-ended questions that help children construct math knowledge, listening to children's stories that include math thinking and vocabulary and scaffolding those by providing appropriate information that relates to the math that naturally occurs in children's play. This research reinforces the way children learn – they are up and around, busy and involved, engaged in doing. The data help define the teacher's role in identifying math learning and supporting that learning, particularly in activities that are initiated by children. Teachers may better understand their role in children's construction of math knowledge – in the middle of the action, not only as a teacher and facilitator but as an active participant. Once teachers begin to recognize the math learning that happens through children's play, teachers will be better equipped to scaffold that learning and help children perceive themselves as mathematicians. Helping children feel confident in math at an early age may help combat math anxiety in the future.

This research may help directors of early childhood programs and school administrators support teachers as they take an active role in helping children learn math through play in nature. The data may reinforce administrators' decisions to allocate ample time in children's daily routine to spend time in outdoor settings, consider what kind of intentionally designed outdoor spaces will best facilitate children's learning, and advocate for the development of those spaces. Children's interests naturally connect them to the world of mathematics. The more administrators understand this, the better they will be able to support the notion that children learn math through much more than rote memorization of content, skill and drill, and a set of prescribed curriculum.

This research may help educational policy makers re-think priorities and examine how to provide the highest quality learning experiences for children. These findings, and subsequent research, may encourage policy makers to revise school policy; to pay attention to learning that occurs outdoors as well as indoors, and to consider funding intentionally designed outdoor spaces to support children's learning in integrated, holistic ways. The children's stories in this research provide real, rich examples of math learning that takes place through outdoor play, and may serve as a resource when policy makers and administrators communicate with the media and other key stakeholders.

The results of this study may encourage math professors to reinforce the importance of children's hands-on play in nature, with interesting materials. It may help them broaden their definition of math manipulatives, to include open-ended materials found in abundance in nature. Our findings suggest that math learning is a process of construction, not merely acquisition of knowledge. Learning often originates in a social setting from children's direct interactions with peers and teachers, during play scenarios. The data may help future math teachers realize that young children develop an early understanding of the ideas and processes of math before they grasp the symbols and rules, and help math professors support experiences in nature that foster meaningful learning. The findings may also encourage college and university professors in teacher education programs to create curriculum that addresses teaching math in outdoor classrooms, and design experiential activities for pre-teachers to help them become more comfortable teaching outdoors. Ultimately this may result in a cadre of new teachers who understand the link between the outdoors, the role of play in children's learning, and the experiential ways children construct math knowledge.

Our data may provide new insights for parents, into ways their children are already literate in math, and how their children develop math thinking and skills through play. The findings may help empower parents to be advocates for their children when they interact with teachers, administrators and school policy makers – about access to quality, intentionally designed outdoor spaces, interesting materials that support rich math learning, and about the kinds of learning experiences schools should be providing for children. The data may encourage parents to spend time outdoors with their children, discovering the mathematical connections inherent in the natural world, and realizing that children use their background and experiences to construct math meaning for themselves.

Procedures

Research Approach

We used a single, case study research approach with an embedded analysis that focused on one subunit of the data (Creswell, 2013; Yin, 2014). Qualitative case studies search for meaning and understanding about a central phenomenon, using researchers who serve as the primary instruments for data collection and analysis. Those researchers collect data through interviews, observations, and the review of relevant documents, artifacts and visual materials. They inductively analyze the data to identify key themes that emerge, and create a written narrative that is rich in description (Merriam, 2009).

Creswell (2013) defined case study research as the study of a bounded system, typically bounded by characteristics of time, place and the specific case being studied. Merriam (2009) similarly defined case study research as “an in-depth description and analysis of a bounded system” (p. 40). This means that the central phenomenon of interest is “a single entity, a unit around which there are boundaries” (p. 40). The case might focus on a person who illustrates a particular phenomenon, a program, a group, an institution, a community, or a specific policy of interest (Merriam, 2009). Stake (1995) suggested that researchers choose to use case study research because a specific case is of interest to them. When researchers are interested in focusing on a case because of what they can learn from that particular case (rather than what they can learn about other cases or about a general problem) Stake calls that an "intrinsic case study" (p. 3).

Merriam (2009) identified a number of special features that apply to case study research. Cases studies are *particularistic*, meaning that they focus on understanding a particular phenomenon. The particular case is important because of the information it reveals about the phenomenon. Case study reports are *descriptive*, and researchers use thick, rich description to present the findings. "Thick description is a term from anthropology and means the complete, literal description of the incident or entity being investigated" (Merriam, 2009, p. 43). Case studies are heuristic, with the goal of "illuminating the reader's understanding of the phenomenon under study" (p. 44). Stake (2007) said case studies illuminate readers' understanding of experience, and make "naturalistic generalizations" that capture researchers' interpretations (p.3).

This case study was particularistic, descriptive, heuristic and intrinsic. The focus of this study was preschool and kindergarten-age children. We were intrinsically interested in this particular case, because we wanted to understand how young children were developing mathematical thinking and skills in a uniquely designed outdoor classroom environment. The boundaries we identified for this study included: a focus on one academic content area (mathematics), a particular age group (preschool and kindergarten-age children), and a particular setting of interest (the research lab school where the first Nature Explore Classroom was constructed and where teachers were trained as co-researchers and asked to document children's learning in the outdoor classroom). Our goal is to use this exploratory case study to help readers understand this phenomenon in-depth.

Yin (2014) suggested that researchers can either conduct a holistic analysis of a case or cases (i.e., the examination of the entire case), or an embedded analysis (i.e., the examination of a specific aspect of a case). Our work represents an embedded analysis. Teacher/co-researchers collected the data we analyzed over several years and those data were part of a larger, ongoing, holistic study at Dimensions Early Education Programs in Lincoln, Nebraska. The initial goal of our larger, holistic case study was to better understand children's learning in the outdoor classroom (in general). Since the outdoor classroom was constructed in 2004-2005, teachers have regularly documented children's activities, interactions and learning in that setting. Over the years we have collected and analyzed data on infants, toddlers, preschool and kindergarten-age children; presented our work at regional, national and international conferences and at research symposiums; and published our findings. For the purpose of this study, we examined our existing database, and conducted an embedded analysis that specifically focused on the mathematical thinking, skills and vocabulary preschool and kindergarten-age children were learning in the outdoor classroom. We also explored the teacher's role in supporting children's development of math skills and the kinds of materials that supported that learning. In the end, as Yin (2014) suggested is characteristic of an embedded case analysis, we selected this one analytic aspect of the larger case for presentation.

Data Collection

The primary data collection procedure teacher/co-researchers used was close observation. Teachers were participant–observers in the setting; i.e., their primary role was teaching and their secondary role was the co-researcher role. As part of that co-researcher role, all teachers were trained on both qualitative research methods and close observation/recording techniques. Since Dimensions Early Education Programs is a lab school for Dimensions Educational Research

Foundation, teachers' daily responsibilities include documenting their observations of children playing and working in the Nature Explore Classroom. Teachers record their observations using a specific template we created (see Nature Notes documentation protocol, Appendix A). Documenting children's learning in the Nature Explore Classroom has been an important part of teachers' roles. However, teachers were not instructed to focus on any one skill in isolation. They were simply asked to record children's behaviors, interactions, and engagement in activities that they believed were significant and provided evidence of important learning that was occurring. Once our research team selected a focus on math, our task was to review teachers' documentation and purposefully identify Nature Notes that illustrated children's development of math skills, thinking and vocabulary.

Initially we reviewed over 500 observations that preschool teachers had recorded between November 2005 and April 2013. During that review we identified 148 Nature Notes that provided rich examples of children's learning specifically in mathematics. Nineteen teachers recorded the 148 Nature Notes we analyzed, and most were experienced teacher/co-researchers. In fact, 13 of the 19 teachers (68.4%) had been employed by the Early Education Programs all eight years these observations were recorded (2005-2013), which is uncommon based on the high turnover rates in early education programs (approximately 30% turnover annually) (Porter, 2012).

Teachers recorded observations during the traditional academic year; from August through May, which aligned with the early education program's calendar year. Sixty-eight (46%) of the Nature Notes were recorded between the months of August and November, forty-eight (32%) were recorded between December and February, and thirty-two (22%) were recorded between March and May. Teachers recorded almost all of the Nature Notes in the Nature Explore

Classroom, with the exception of a few that were recorded near the parking lot adjacent to the outdoor classroom, on the sidewalks or block around the building and in the indoor classroom (specifically when the observation was a direct extension of outdoor activities).

We also conducted one hour-and-a half-long focus group interview with the teachers. Eighteen teachers participated in the interview, and to keep the numbers manageable for focus group interviews, we interviewed three groups of teachers, five to seven teachers per group. We conducted the interviews over a one-week timeframe. We created a semi-structured interview protocol with questions that were exploratory and open-ended. We gave teachers the questions in advance, and asked them to record their thoughts in preparation for the interview. The protocol included one over-arching question: In what ways have you observed children developing math skills and constructing math knowledge in the Nature Explore Classroom? Three probes encouraged teachers to reflect on this question more specifically: 1) What math-related activities do you see children doing, 2) What natural materials lend themselves to math learning, and 3) Can you share some specific examples of children learning math concepts in the outdoor classroom? We also asked teachers in advance to bring written observations to the interview that they believed specifically illustrated children's learning in mathematics. During the interview we asked teachers to share their documentation – to discuss what children were doing, the materials they were using, and the math learning that was happening. In addition to examining teachers' observations and analyzing focus group data, we reviewed visual materials teachers submitted with their Nature Notes, including sketches they made, children's writing/drawings/artwork and photographs teachers had taken. At the end of the interviews, 14 of the 18 teachers submitted the notes they had recorded in preparation for the interview, and we included those notes in our analysis. Table # 1 provides a visual overview of our data.

Table # 1: Characteristics of the Data (Teacher/Co-Researcher Observations)

Characteristic	**Frequency and/or Percentage**
Number of observations purposefully selected for embedded analysis	148
Focus group interviews	One 1.5 hour interview with 18 teachers November 2008
Date range observations were recorded	November 2005 – April 2013
Number of teacher/co-researchers who recorded observations	19 (13 of 19 (68.5%) taught all 8 years)
Months during which observations were recorded	August through November – 68 (46%) December through February – 48 (32%) March through May – 32 (22%)
Number of observations that represented teacher-initiated activities vs. child-initiated activities	Teacher-initiated – 34 (23%) Child-initiated – 114 (77%)
Areas in the outdoor classroom where observations were most frequently documented (Note: Some observations were recorded in multiple areas as children moved around the outdoor classroom)	Messy Materials Area – 35 Greenhouse including raised beds – 30 Sand – 17 Garden and garden pathways – 17 Gathering Area – 14 Open Area – 8 Climbing/Crawling Structure – 6 Nature Art Area – 6 Dirt-Digging Area – 5 Block Building Area – 4
Number of observations where teachers recorded math vocabulary (from simple to more complex language)	125 (84%)
Use of purposeful movement/motor skills recorded	Whole body/gross motor – 97 (66%) Fine motor – 71 (48%)

Data Analysis

Our research team created a protocol (in tabular form) to assist in systematically analyzing teachers' Nature Notes. We read through all of the documentation multiple times and initially spent, on average, 30 minutes carefully extracting relevant data from each Nature Note. As we reviewed teachers' observations we recorded the following on the analysis protocol:

- The date the observation was recorded;
- The name of the teacher/co-researcher who had documented the observation;
- Whether the interaction or activity documented was teacher or child initiated;
- Children's names, ages and gender;
- The context of the observation (i.e., what children were doing);
- The area(s) in the Nature Explore Classroom (or nearby) where the documentation was recorded;
- The math skills/concepts children were using/developing/practicing/physically experiencing;
- The math vocabulary children were using (if teachers specifically recorded this using quotation marks);
- The materials children were using that supported their math learning;
- Whether or not children were engaged in purposeful movement (and whether that movement was whole body/gross motor or fine motor);
- The teacher's role in supporting children's math learning (when teachers specifically documented their actions and words on the observation protocol); and
- Notes about visual materials attached to the documentation.

After we examined all 148 observations, the visual materials and the focus group data, we organized our findings around four thematic constructs to create the written narrative. We discuss these themes in the Findings section.

Findings

Introduction to the Site:

Dimensions Early Education Programs is located in Lincoln, Nebraska in an older, residential neighborhood that is rich in architectural beauty, with established trees and beautiful parks within walking distance. Its infant, toddler, preschool and summer Kindergarten – Grade 5 programs serve as Dimensions Educational Research Foundation's research classrooms. Dimensions began as First-Plymouth Preschool in 1967, and became a 501(c)3 organization in 1998. The Early Education Programs are housed in First-Plymouth Church, a spacious, almost 80-year-old brick building with intricate architectural features. However they are not affiliated with a religion and welcome children from all backgrounds and income levels. Dimensions employs 23 teachers (with an average tenure of over 10 years), four administrative staff, and serves between 250 and 350 children each year, including the summer programs.

In 2004–2005, Dimensions constructed an outdoor classroom for preschool, kindergarten and school-aged children (later a separate outdoor classroom was constructed for infants and toddlers near their indoor classrooms). The preschool outdoor classroom was the third Nature Explore Classroom certified (in May 2007). The space is approximately 82' x 78' and includes several specifically designated areas. Children meet at the Gathering Area to make their plans for their time outdoors. A 12' x 20' Greenhouse, with child-size tables and benches (constructed by a parent out of rainforest wood that would handle the moisture), gives children the opportunity to interact with plants year-round. The spacious L-shaped Sand Area allows an entire class of children to play in the sand at one time. The large, Messy Materials Area, filled with natural materials to manipulate, such as sticks, stumps, woodchips and sections of cut tree trunks (i.e., “tree cookies”) is bordered by a low (child height), organic-shaped brick wall, which was created by a local sculptor. Children went to the sculptor’s studio to learn about the kiln and brick-making process and to put their handprints and footprints in the bricks before they were fired. Later the sculptor filled several of the depressions with melted glass. Next to the Messy Materials Area is a separately designated area, with a hard (tree cookie) surface to provide stability for building.

Three children, ages 11 and 12, who were alumni of the preschool program and volunteered in the summer program, designed the rectangular-shaped Dirt-Digging Area, which is located next to the Block-Building Area. They worked with a local sculptor to design the brick border around the space, laying it out with large cardboard blocks in one of the classrooms prior to constructing it, calculating the dimensions and how many bricks they would need to build it. The wide brick border, recessed into the ground, provides children with places to sit as they dig.

A towering climbing-crawling structure with multiple slides, ladders and landings gives children the opportunity to physically experience different levels and exercise their whole bodies.

A large Perennial Garden (approximately 15’ x 36’) spans the north end of the Nature Explore Classroom, with limestone paths through it. It contains prairie plants that can withstand dry, hot summers. The plants were intentionally selected to provide a variety of colors, shapes, smells, sizes and textures; to attract insects; and because they had interesting stories associated with them. A key goal was to provide children with rich, multi-sensory learning experiences through their interactions with nature. The seeds and bulbs children plant in the Greenhouse are often transferred to the garden where children can watch them grow, care for them, harvest them, and taste the edible plants.

The Vegetable Garden is approximately 17’ x 28’ and has been terraced into multiple levels, with pathways so children can access every tier. Children love climbing the stairs and experiencing the level changes and watching the garden grow. Children have grown sweet corn, multiple varieties/colors/shapes/sizes of tomatoes, pumpkins, squash, kale, basil, garlic, carrots, green beans, purple potted pole beans, cabbage, lettuce, a variety of peppers and okra in the garden. In the past, children have had the opportunity to share the produce they grew during an Farmers’ Market that brought parents and the community to the Nature Explore Classroom.

A large, grassy Open Area in the middle of the Nature Explore Classroom provides space for children to engage in whole body, gross motor movement activities. A corner of the open space is designated as a Music Area. It includes an akimbira made from Brazilian hardwood (harvested sustainably) and teachers bring drums and other natural instruments outdoors daily.

The artist’s garden (Nature Art Area) was recently renovated to provide a more beautiful, inspirational, and functional space for children to create art outdoors. Children helped design the multi-tiered, mosaic surface where easels stand. They broke glass dishes and ceramic tiles for the flooring, and created their individual designs by arranging glass pieces on trays. Dimensions’ art specialist and a sculptor incorporated children’s designs into the mosaic surface by transferring them from the trays to the adhesive base.

In the northeast corner of the Nature Explore Classroom (completed in November 2009), at the edge of the artist’s garden, is a smooth, child-height corner bench, large enough to hold a group of children. In contrast to the bench, positioned directly behind it on the other side of a wrought-iron fence, stands a taller, ruggedly textured wall, with sculpted bricks placed vertically and irregularly, with occasional small openings. These negative spaces were intentionally incorporated to provide children with multiple perspectives as they peek through the wall to the world outside. The smooth brick bench matches the architectural features of the low brick wall that borders the Messy Materials Area and the rugged wall has similar depressions (made by teenagers' hands and feet), with melted blue glass in them. A small strip of mosaic runs through the smooth seat of the bench, visually tying it to the artist garden's mosaic floor.

Children spend time daily in the Nature Explore Classroom, and on a typical day, visitors might see children:

- making a plan about where they choose to play;
- initiating games and creating pretend scenarios;
- using their imaginations to transform natural materials into food, houses, castles, rocket ships, roads, ice cream stands, tools, and equipment;
- pretending to be someone or something else (e.g., birds, animals, insects, plants, community workers, family members);
- engaging in dialogues related to their play "scripts;"
- exercising their large muscles as they move freely through the pathways and Open Areas of the outdoor classroom;
- negotiating and collaborating as they work together to accomplish a goal;
- solving problems about how to move and manipulate heavy natural materials;
- proposing and testing hypotheses about how things work;
- sharing their knowledge and experiences with each other;
- rehearsing and practicing newly acquired skills;
- learning through trial and error and repetition, and most importantly,
- having fun!

Themes

Four key themes emerged as we analyzed the data. The first theme identifies the key math skills young children developed as they interacted with the environment, each other, teachers and materials. The second theme describes the math vocabulary children used in the context of their play (only noted when teachers specifically documented vocabulary using quotation marks). The third theme identifies the kinds of materials in the Nature Explore Classroom that contributed to children's math learning (natural, living, added materials and tools). The fourth theme examines the teacher's role in supporting children's math learning. Table # 2 provides a visual overview of the key themes and sub-themes.

Table # 2: Key Themes and Sub-Themes

Themes	Sub-Themes
Key Math Skills Children Were Developing	• Classification • Estimation • Geometric Shapes • One-to-One Correspondence/Number/Counting • Whole-Part Relationships • Making Comparisons • Measurement • Scale Relationships • Sequencing/Seriation • Volume • Time Concepts • Computational Skills • Matching • Fractions
Math Vocabulary Children Were Using	• Classification–Identification/Sorting • Computation • Measurement • Number/Counting • Quantity • Size Relationships • Time Concepts
Materials That Supported Math Learning	• Natural/Open-Ended/Loose Materials • Living Organisms

	• Added Materials and Tools
The Teacher's Role in Supporting Math Learning	• Asked Questions to Encourage Math Thinking • Provided Materials, Information and Vocabulary That Supported Math Learning • Assisted Children With Activities That Supported Math Learning • Requested/Invited Participation in Experiences That Supported Math Learning

As we constructed the findings section the relationship across our data became apparent. We realized that children do not construct math knowledge in isolation from other disciplines. As we discovered in an earlier study on authentic play in a Nature Explore Classroom (Miller, Tichota & White, 2013), when children spend time in the outdoor classroom, they are developing skills in multiple domains simultaneously (in the 63 observations we analyzed for that study, children developed skills in five to nine domains simultaneously, p. 79). Readers will discover in the stories we share in this paper that not only were children learning about math, but they were learning about science, developing social skills, honing their visual-spatial skills, developing language and literacy skills (written and verbal), learning about construction and engineering principles, developing important intrapersonal skills (e.g., initiative, self-confidence, executive functions like planning and decision-making), exercising their imaginations through their symbolic, creative representations, and developing body competence through the physical experiences they were engaged in outdoors (both gross and fine motor).

Qualitative researchers present data as evidence of their emerging themes and working hypotheses. Selecting the stories to share in this paper was not difficult; we had so much rich data. Deciding which theme those stories best supported was sometimes challenging because the themes are inextricably related. For example, we included several examples of documentation that illustrate the teacher's role in supporting children's math learning. The same Nature Notes that beautifully illustrate the teacher's role also describe specific math skills children were developing, the ways they were using interesting materials, and often include math vocabulary. We hope that these connections help readers understand the holistic nature of children's rich learning experiences outdoors.

Key Math Skills Children Were Developing in the Nature Explore Classroom

The first theme, and primary focus of this research, is the math skills young children were developing through their play experiences in a Nature Explore Classroom. The outdoor classroom was intentionally designed to support children's learning through play. Our data suggest that children were constructing math knowledge through their observations and interactions with the rich built and natural environment, and through their interactions with their teachers and peers. Many of the Nature Notes we highlight in this paper represent shared experiences. Children learned from each other as they shared their knowledge with one another, asked questions, and worked together toward a common goal. The lessons children learned were not always "measurable," yet it was important to their development of early math understanding that they had opportunities to physically experience math principles through their play. Because many of the observations were child-initiated, children were highly motivated to pursue their plans and driven by their natural curiosity about the world and about math.

The data (i.e., stories) we share in the findings section were drawn directly from teachers' written documentation (close to verbatim, with slight editing, and for the sake of consistency we removed personal pronouns when teachers wrote themselves into the story). In this section, we include the teachers' Nature Notes, and at the end of each one identify the key math skills children were developing. Tables at the end of the section more comprehensively identify (from our larger dataset) specific ways children were constructing math knowledge through their play, practicing math principles, and honing their emerging math understanding.

Sketching Flowers – From "Smallest to Biggest"

Lana and her friend Ellie (both five) were in the Greenhouse with Mrs. Guess, talking about what they should draw. Lana decided she was going to draw flowers. She'd had lots of experiences observing flowers in the garden and Greenhouse. She began her sketch with the ground and with a seed. She added four flowers of different sizes, from right to left at the bottom of her paper. Each was a different height and as the flowers became larger, so did the leaves. She described her picture to Mrs. Guess as "the smallest to the biggest." She knew the smallest needed to be a seed and then her illustration showed how the seed grew. Her first flower (next to the seed) just had leaves and then they started to get bigger. She illustrated the progression in size as the plant grew. She also added a sun and a cloud at the top of her paper. She said the sun represented "what the flower needed." Her cloud was "for rain to help the flower grow." She also added herself in the picture (Heather Guess, March 2009).

Key Math Skills: Seriation (smallest to largest), Symmetry (of leaves), Math Vocabulary, Translating prior 3-dimensional observations to a 2-dimensionsl medium, Spatial relationships

Roasting Marshmallows

Two boys, Nick and Josh (both almost five), were in the Sand Area pretending that they were roasting marshmallows. Mrs V. joined into their play. Several bricks formed a circle (made by other children), which Nick and Josh decided made a perfect fire ring. As they sat round the "fire." Mrs. V. asked Nick if he knew how many bricks were in the circle. He touched each brick as he counted aloud and answered, "Ten!" They decided to make s'mores! After they ate their “s’mores,” which they made using chunks of small tree cookies, Mrs. V. asked Josh to draw a picture of the s’mores for her. He drew them on half of her documentation form, and his drawing included lines and geometric shapes (Kris Van Laningham, January 2012).

Key Math Skills: Counting, One-to-one correspondence, Translating 3-dimensional experience to a 2-dimensional medium, Spatial relationships, Geometric shapes

Stringing Corn and Cranberries to Decorate the Greenhouse for the Holidays

During a visit to the Greenhouse children were working with corn kernels and cranberries that the school’s garden specialist had soaked in water and brought to school. She told children that they could create something to decorate the Greenhouse for the holidays. She gave each child a needle and thread so children could string them. Children first laid out their corn kernels and cranberries on the table, in an alternating pattern. They used their fine motor skills to carefully line up their needles with the center of the cranberries and pushed hard to get the needle to go all the way through. They were careful not to poke their fingers as they pulled each cranberry and kernel of corn down the length of the thread. As they worked, the teacher talked with them about the size of the corn plant (“a big plant with tall leaves”) in relation to a small corn kernel. They looked closely at the silk and tassels of a corn plant and held corncobs that had hard corn on them (Tina Reeble, November, 2005).

Key Math Skills: Creating an alternating pattern, Identifying center, Whole-part (in discussion of corn), Length (of thread, and string), Whole-part relationships (corncobs and kernels)

Creating and Sketching a Path (and counting the “steps”)

Five-year old Cami worked mostly by herself in the Messy Materials Area. At first, she walked back and forth on the low brick wall that serves as a boundary between that area and the

Open Area. Soon Cami began collecting short, round tree stumps and large cubed-shaped tree blocks. She carried them over, near the wall, and lined them up (in a curved line), creating a pathway that led toward the brick wall. After she arranged the wooden blocks and stumps, she noticed her teacher sketching. Cami asked if she could "draw" her path. Mrs. Jensen provided paper and a pencil and Cami drew the path she had made on the paper, representing the stumps using circles and the cubed blocks using squares. She hopped several times along the path she created, and continued to count each time she stepped on a piece of wood. Mrs. Jensen discovered that Cami was checking the number of pieces of wood she had drawn on her paper with the actual number of stumps and cubes on her path. She came back to her paper multiple times to adjust her drawing. When she finished she handed her paper to Mrs. Jensen. Cami had accurately drawn the two shapes she used for her path (as well as the curve of the line/path) and as she checked her work she had crossed through (with an X) each piece of wood she had counted. Cami told her teacher, "The ones I crossed off are the right ones" (Barb Jensen, September 22, 2008).

Key Math Skills: Counting, One-to-one correspondence, Translating a 3-dimensional experience to 2-dimensional medium, Geometric shapes (in sketch)

Reaching High to Touch Tree Branches

Two five-year old boys stood under one of the large evergreen trees planted in the Messy Materials Area. They wanted to see if they could reach the branches, but their arms were not long enough. Camden picked up one of the long, 6' bamboo poles, and used it as an extension to his body, pushing it up toward the limbs as he observed the height of the tree branches. He said to his friend, "Look how big this stick is. It's five or fifty feet." His friend Edward replied, "It's big enough to touch the tall tree leaves." (Joyce White, December 7, 2008).

Key Math Skills: Number, Estimation, Height (with arms and extension), Math vocabulary, Physical experience of both vertical and horizontal

Creating a Map of the Nature Explore Classroom

During outdoor time children were sketching in the flower garden. Ms. Murdoch suggested that they make a map of the garden, but none of them seemed interested or responded to her idea. Later, she noticed five-year old Melissa, sitting on a low wall at the bottom of the flower garden. When she got close enough to see Melissa's paper she discovered Melissa was drawing a map of the outdoor classroom. Besides the garden, it included the Climbing/Crawling Area on the opposite side of the large space, the Greenhouse, pathways and landmarks. Melissa worked intently for 30 minutes, and then shared her very detailed map with Ms. Murdoch. Most of the areas she depicted on her map were accurate in their relative positions. As they walked together through the space, Melissa explained what she had drawn on her map, while Ms. Murdoch documented those for her (Holly Murdoch, May 25, 2006).

Key Math Skills: Translating 3-dimensional to 2-dimensional, Geometric shapes, Lines, Spatial relationships

Collecting and Classifying Nature Items

Emma had collected many items from the Nature Explore Classroom. Using two recyclable frozen meal trays, she created a hinged container to carry them (by taping one end together). When Emma's class went indoors, she sat on a chair and all the children gathered around her. Before she told her friends what was inside the container, they tried guessing. Caden said, "Acorns…A hundred of them, maybe a million." Kallie guessed butterflies: "I think there are ten." Mercedes guessed snakes: "Maybe five." Then Emma opened her container and allowed her friends to touch her treasures, one at a time. Once her friends were finished examining the contents of her container, she sent them to wash their hands for lunch. After lunch, Emma sat down with her collection, two pieces of blank, white paper and colored markers. She categorized the items she had collected. With Mrs. White's help they placed the items on both pieces of paper (in rows). Emma counted eight different categories of natural materials that she had collected. She wrote the numbers one through eight on her papers, by each row of items. Row one had pieces of milkweed pods (laid end-to-end) and a seed. Row two had a dried up center of a small sunflower, with a partial stem on it (Later when she sketched it on a documentation form she said, "It has a triangle."). Row three had a line of four acorns plus two lids off acorns. Row four had two leaves, with different shapes, laid end-to-end. Row five had a single small, dried up pinecone. ("It reminds me of a raindrop," she said as she rolled it back and forth between her fingers. "Hey, look…there's a hole in the bottom. Look at this hole in the middle. It's kind of like a cave.") Row six had a single dried up, wilted flower. Row seven had a piece of a butterfly wing ("I wonder what kind?" she said. "It doesn't look like a Monarch."). Row eight had a lot of small pieces of woodchips. Emma also sketched her collection (Joyce White, August 27, 2012).

Key Math Skills: Classification, Number, Translating 3-dimensional to 2-dimensional, Lines, Geometric shapes, Math vocabulary, Whole-part relationships

Pizzas For Sale!

Maddie and Eli were playing in the Messy Materials Area and decided to "sell pizzas." Maddie announced: "We have a restaurant and I am the owner!" Mrs. Miller asked, "What kind of restaurant?" and Eli said, "Pizza." Maddie asked the teacher, "What is the first letter of your name?" and Mrs. Miller said, "My first letter is M." Maddie had a clipboard and paper and wrote "M" on her paper. She said, "Mark wanted a pizza, so we have two M's." Eli told Mrs. Miller, "Here's your pizza." Maddie began calling customers over to the Messy Materials Area where her restaurant was located. "We have pizza here, we have pizza here!" She took orders for pizzas, and Eli prepared and delivered them. When Lara wanted to buy a pizza, Maddie asked, "What is the first letter in your name?" and Lara replied, "L." Maddie had drawn circles on her paper to represent each of the pizzas she sold and above each circle ("pizza") she put the first letter of each customer's name. Mrs. Miller asked, "How many pizzas have you sold?" Maddie counted the circles on her paper: "1, 2, 3, 4, 5, 6, 7, 8, 9." "Wow," Mrs. Miller exclaimed, "You've sold nine pizzas!" Maddie said, "Almost 10." Sara ordered a pizza and Mrs. Miller asked, "How many pizzas have you sold now?" Without re-counting, Maddie said "10." Eli counted too. Mrs. Miller asked Maddie, "How did you know how many pizzas you sold without counting?" "Because my Grandma teaches math," she explained. She continued to sell pizzas. Martin and Cinda ordered pizza. When Maddie asked Martin, "What's the first letter in your name?" he replied, "Martin." Maddie said, "I don't need your whole name. Just the first letter." Gina came to order a pizza and when she told Maddie that the first letter in her name was "G" Maddie asked, "How do you make a G?" Gina said, "Like this" and made a G in the air with her finger twice. "Do you know how now?" she asked Maddie (Sherry Miller, November 28, 2011).

Key Math Skills: Classification, Number, Counting, One-to-one correspondence, Addition, Translating 3-dimensional to 2-dimensional, Geometric shapes

A "Seed Scientist" Exploring Seedpods

Gary sat in the sun at a small picnic table outdoors with Miss Allie, a student teacher. He had collected locust seedpods from the Messy Materials Area. As he gave Miss Allie some seedpods he said, "These are mine and those are yours." He had discovered that if he broke the pods apart, there were seeds inside. They each opened their pods and put the seeds they found inside in two piles. When Miss Allie ran out of seedpods she asked Gary if she could have some of his. "No," he replied, "but you can go get some more." She did and Gary and Miss Allie broke open seedpods for at least 15 more minutes. Gary told Miss Allie that she could take her seeds home to plant. At the end of their outdoor time, Gary walked up to his teacher, Mrs. Tichota, and said, "I'm a seed scientist. I'm going to take these seeds home and plant them." He showed his seeds to the other children in his group. (Mrs. Tichota later asked Miss Allie if she had called Gary a "seed scientist" and she said she had not.) (Kathy Tichota, January 18, 2013).

Key Math Skills: Classification (locust seedpod), Quantity, Whole-part relationships, Dividing quantity into two piles

What's Your Preference? Interviewing Friends

On a 29 degree February day, four-year old Dallas went to the Greenhouse and gathered a clipboard, paper and pencil. He drew several large square and rectangular shapes (and one triangle) on his paper and announced to anyone who would listen, "I am going to interview people." He first found Luke in the Open Area and asked, "What do you like best – do you like to shovel or eat ice cream?" From their brief conversation Dallas concluded, "I think he likes eating ice when he shovels." Then he asked Luke, "Do you like to run fast?" When Luke replied Dallas repeated the words, "run fast" as he wrote the letters T, F and H inside one of the squares on his paper. He continued to interview several children who were outdoors, and each time they replied to his questions he wrote letters inside one of the boxes he had drawn on his paper. (Joyce White, February 27, 2013)

Key Math Skills: Geometric shapes, Spatial relationships, Math vocabulary, One-to-one correspondence (individual responses represented in child's sketch)

“Shelling corn, shelling corn” – “Grind it up, Grind it up!”

Ms. Murdoch brought a manual grinder to school so children could grind corn. They worked in the Greenhouse, with ears of Indian corn with the husks still attached. They took the kernels off the corncobs and put them in the grinder. Once the class was finished, one four-year old boy, Alex, stayed longer so he could continue the activity. The teacher worked individually with him for another 15 minutes. He talked with her the entire time, almost stream of consciousness. He asked the teacher what the ground corn was for, and she replied, “What do you think?” He said he thought it was for the birds and squirrels, and added: “Sometimes squirrels and birds have to eat plain old corn.” Alex told his teacher a story as he recalled an earlier experience with corn: “One time I took corn off the cob at the apple farm. We ate lunch there.” At one point in the conversation he observed, “There’s a moth in here and I think it’s yellow.” He randomly added, “I don’t wish I was a hunter shooting a bird. I would be sad for the bird.” He told his teacher: “This is very fun. I could stay here for the whole hour. What time is it?” (Teacher: “About 3:45.”) As Alex worked he made up repetitive songs: “Shelling corn, shelling corn,” sung at one pitch, then at a higher pitch, again, “Shelling corn, shelling corn.” Then “Grind, grind it up; Grind, grind it up. Grind, grind it up, to see what it turns like.” When the grinder was full, Alex dumped the powder (which he called “dust”) into a glass jar, using a funnel. He noticed that the grinder was getting empty and told Ms. Murdoch: “When the corn is deep down low, it’s hard to cut. Maybe it needs some more.” Alex asked some questions about the husk; how to take it off, and what would happen if they put that in the grinder. After Ms. Murdoch answered his questions they went back indoors and joined the rest of the class (Holly Murdoch, September, 24, 2009).

Key Math Skills: Whole-part relationships, Volume, Time concepts (recall), Math vocabulary, Classification (color), Repeated pattern

Lines! “Come see what I made!”

On an autumn day in the outdoor classroom Hayden, age three, approached his teacher enthusiastically: “Come see what I made!” He showed her a thick line in the woodchips in the Messy Materials Area. She asked him, “How did you make that line?” Hayden replied, “With my bottom,” and illustrated by sitting down and scooting through the woodchips and pine needles. A few minutes later, Hayden continued his exploration of lines. He found a stick, which he pulled through the woodchips surrounding the Climbing/Crawling structure. This time, instead of the wide path his body made in the chips, Mrs. Tichota noted that he made “long, skinny lines.” He was not quite finished exploring lines, so he used a pencil and piece of paper to draw the lines he had made with his body and the stick. He drew three vertical lines, parallel to one another, almost the length of the paper (Kathy Tichota, October 9, 2008).

Key Math Skills: Lines (Length, Width, Parallel), Translating 3-dimensional to 2-dimensional

“Look teacher, I made this of flowers!”

Four-year old Luke went into the Greenhouse and picked up a clipboard with paper and a crayon. He worked independently until he went to his teacher and said, “Look teacher, I made this (picture) of flowers.” He had been closely observing the flowers in trays in the Greenhouse, and drew his interpretation of them. Rather than a literal depiction of flowers, Luke had drawn four quadrants, with arrows facing different directions in each quadrant. He showed Mrs. White his sketch and explained, “This flower faces this way (representing the direction with the arrow), this flower faces this way, this flower faces this way, this flower faces this way…" repeating this phrase each time he pointed to an arrow (Joyce White, March 21, 2007).

Key Math Skills: Translating 3-dimensional to 2-dimensional, Use of symbols/arrows/quadrants, Directionality

"A Log Experiment!"

Five-year old Paul conducted "a log experiment." He moved a long log from the Messy Materials Area to the Climbing/Crawling structure. The log had different sizes of nubs sticking out where branches had once been. He told Mrs. Tichota, "It has handles and a little fuzz." She asked Paul, "What else do you notice about the log?" She had sketched the log on her documentation form and Paul looked at her sketch. She had drawn the two longer "handles" but not the short nub. He noticed and said, "Draw the little nub." Mrs. Tichota added it to her sketch as Paul continued: "The ends are curved a little bit, they're not straight." Paul used his hands to demonstrate the words "curved" and "straight." He said, "I'm doing a log experiment" as the teacher was called away to take care of another child. Paul later told Mrs. Tichota that he sent the log "down the slide and it went 100 miles per hour" (Kathy Tichota, November 28, 2012).

Key Math Skills: Weight, Math vocabulary, Lines (curved/straight),

Speed **Creating an Obstacle Course**

When they went outdoors, Grant, Maddie, and Matt (all four) noticed numbers written on chunks of wood with chalk, done by children in a previous class. Mrs. V. asked if they would like to make an obstacle course with wood chunks. They worked hard, hauling large, heavy chunks of wood to the Messy Materials Area, and arranged them in a curvy line (working to stabilize each piece). Mrs. V. asked if they wanted to write numbers on the chunks of wood with chalk to show the direction of the obstacle course. They wrote the numbers 1 through 29 consecutively on the chunks of wood. Once they finished constructing their course, they and many other children walked it and counted as they stepped on each cube. Maddie carefully studied each wood chunk so she could write the numbers on a piece of paper. She wrote all of the numbers she saw on the wood chunks on a piece of paper (Kris Van Laningham, March 23, 2011).

Key Math Skills: Number, Counting, One-to-one correspondence, Lines (curvy)

Finding Roly Polys!

Mrs. V's morning class was planning to go to the Greenhouse to collect roly polys from the worm bin. Harry wanted to finish watching a video about cranes with another class but was torn because he really wanted to go to the Greenhouse. So Mrs. V. told Harry they could go back to the Greenhouse in the afternoon and search for roly polys, so he could stay and finish watching the video. The morning group of children did not have much success finding roly polys – they only found 25. When the class went outdoors that afternoon, Mrs. V. challenged them to "break the record of 25 roly polys" gathered that morning. The children took the challenge very seriously. Another teacher (Ms. Murdoch) was in the Greenhouse when the children arrived and as the children searched, she opened a container of soil with lots of roly polys in it, and added them to the "playground" (a plastic, domed roly poly playground Mrs. V. had purchased at a science conference). Once children were finished picking out roly polys from the worm bin and putting them into the playground, they (along with Mrs. V.) dumped them all out into a container and counted them as they gently put them back in the playground. The children became excited when they counted 105 roly polys! Ms. Murdoch suggested the class put the roly polys on display for other classes to see. Harry made a sign with the number of roly polys that would be on display in the playground (105). They put the roly polys/playground on the picnic table and went indoors. Another class went out and the group was fascinated with the roly polys. They observed them for a long time (up to 15 minutes). Later the children put the roly polys and the sign in the hallway indoors, for other children and the parents to see as they left for the day. Mrs. V. returned the roly polys to the worm bin later in the day (Kris Van Laningham, March 21, 2011).

Key Math Skills: Counting, Number, Classification

"A Race for 21 People"

Oliver (almost five) made a race track by pushing a large push broom through the woodchips around the climbing/crawling structure until he could see the dark, uniform surface underneath. He said he was making a "track…a race track." Mrs. Kelly asked if he would like to make a sign for the track when he was finished. He wanted to and she asked what the sign should say. He replied: "Race. This is a race for 21 people." Oliver wasn't sure how to make the letter "2" in "21", so Mrs. Kelly helped him by making dots in the shape of the number two on his paper, and he connected the dots. She also wrote the word "People" on a separate piece of paper so Oliver and his friend Andy (who had joined in) could copy it onto Oliver's sign. They took turns writing the word "People" on the sign. They taped their sign to a tree and announced the race. The children and Mrs. Kelly lined up for the race and everyone began running around the track. Oliver was not ready for the race to begin and interjected: "This is a practice run." He said, "I'll tell you when the real race is going to start." He told Mrs. Kelly he wanted to wait until there were 21 people to race, like the sign said, but she pointed out that though there were 21 people outside, from two classes, if everyone ran there would not be any spectators. Oliver settled on 13 runners and 8 spectators, and they all counted off 13 together. Mrs. Kelly was walking through the children and Oliver said, "Mrs. Kelly, start at the starting line please." She joined the line of runners and Oliver looked at his watch and said, “On your mark, get set, “ (and pushed a button on his watch that made it beep). The children and Mrs. Kelly ran several laps around the track as other friends cheered and waved. As they were running, Oliver yelled, “If you hear the beep again, the race is over.” He watched them run, then pushed the button again to signal the end of the race. The group ran three more races the same way (Amanda Kelly, September 30, 2011).

Key Skills: Time concepts/vocabulary, Number, Counting, Repeated pattern

The 17 Nature Notes we shared here paint a picture of children developing early math skills while they were engaged in meaningful, primarily self-initiated play. Readers had the opportunity to hear teachers’ and children’s voices and vicariously share their experiences. Tables 3 through 16 provide a more comprehensive picture of our data. We examined all 148 Nature Notes and selected several illustrations of skill development to represent on each table. We organized the data into the following skill categories:

• Classification (Table 3)

• Estimation (Table 4)

• Geometric shapes (Table 5)

• One-to-one correspondence (Table 6)

• Whole-to-part (Table 7)

• Computation (Table 8)

• Fractions (Table 9)

• Making comparisons (Table 10)

• Matching (Table 11)

• Measurement (Table 12)

• Scale relationships (Table 13)

• Sequencing/Seriation (Table 14)

• Volume (Table 15)

• Time concepts (Table 16)

Table # 3: Sampling of Classification Skills

Child(ren) demonstrated classification skills by:
• Finding edible green beans and tomatoes in the garden. • Collecting only ‘small’ rocks. The smallest rocks represented ‘seeds.’ • Finding types of plants and leaves. • Investigating animal tracks in the snow/determining the type of animal. • Collecting acorns by size and quality (whole vs. broken). • Finding and closely observing all the rosemary plants in the Greenhouse. • Observing the color of tomatoes/identifying two different types of tomatoes. • Selecting specific natural materials for building a structure (e.g., sticks, tree cookies, stones). • Identifying insects (cricket, roly poly, centipede). • Identifying attributes of chickadees and juncos (name, type, size and color). • Searching for bugs in the Nature Explore Classroom/classifying four types (ladybugs, bees, grasshoppers and praying mantis). • Identifying living and dead plants on the Nature Explore Classroom/distinguishing which plants to deadhead. • Digging in the Greenhouse worm bin/identifying red bugs, roly-polys and worms.

- Selecting one type of plant to cut (zinnia).
- Observing leaves while picnicking under a tree and going on a nature walk. Child classified color and shape of leaves (red leaf, yellow leaf).
- Identifying all the fall leaf colors in September.
- Trying to identify an unknown insect by specific attributes of size and color. "It's too big for a roly-poly, …too brown for a worm."
- Sorting three items in the worm bin (worms, bugs and sprouted seeds).
- Searching for large, in-tact chunks of ice and snow.
- Collecting, separating and counting specific nature items (e.g., milkweed pods, leaves, acorns, dried-up pine cones).
- Identifying two body parts of an insect (legs and antennae).
- Sorting wood chips and leaves and labeling the wood chips "old money" and the leaves "new money."

Table # 4: Sampling of Estimation Skills

Child(ren) estimated:
• A zinnia's growth over time. • The distance between stumps and tree cookies. • The size of sticks needed to cross the letters H & A. • The depth of the snow and size of sticks in order to allow a stick to stand vertically in the snow. • The size of blocks of wood needed to stop the flow of water at the end of a trench a child dug. • The amount of dirt needed to fill pots when planting.

- The space needed to maneuver 6-foot poles while building around a tree.
- Where to find specific objects covered by the snow, using visual memory.
- The number of stones needed to form a circle to make a "campfire" and how far apart to space the stones.
- The distance for bridging tree stumps (selecting a plank long enough).
- The number of tree cookies needed to stack on top of a stump, in order to create enough height while standing on top to touch a tree branch (added and subtracted cookies, tried variations of 2, 3, 4 tree cookies).
- Volume and distance while working to transport a heavy stone. Child found the right size basket to fit his needs. He emptied the basket of scarves and placed the heavy stone inside, experiencing volume. Then, he estimated the distance between two handles to find a stick long enough to span both handles for carrying.
- The weight and size of birds. While building a birdhouse the child considered how sturdy the perch should be to hold the weight of a bird. Then he considered the size of the hole so the bird could get inside.
- The height and angle to prop long poles to make a pyramid shaped structure children could get inside.
- The depth of a hole they dug (in inches, using a ruler).
- The number of nature items collected to make a home for a ladybug.
- The number of marigold seeds they had collected while dead-heading plants.
- By making a "scientific guess" (at the teacher's request) regarding how many seeds they had removed from hyacinth bean pods.

Table # 5: Sampling of Skills Using/Identifying Geometric Shapes

Child(ren) used/identified geometric shapes when:
• Mapping the Nature Explore Classroom Areas (circles, oval, rectangles, squares and triangles). • Creating an eight-inch solid sphere by packing snow. They labeled it "globe" and colored continents and water on it. • Creating a design proposal for buildings and houses in the Sand Area. They sketched the design symmetrically and used circles, triangles and rectangles. Also, created repeated patterns using diagonal lines and repeated shapes to represent shingles on the roof. • Forming a circle using oval-shaped stones. • Drawing bugs they found and classified earlier outdoors. The teacher extended an outdoor activity indoors including a discussion on possible shapes used for bugs. (e.g. ladybug is shaped like an oval). Children practiced sketching geometric shapes before they created bugs using the shapes they had identified. They also represented whole-to-part while sketching bodies, symmetry and number as they added two eyes, two antennae, and multiple legs or spots on the bugs' bodies. • Sketching lettuce while observing the lines, circles and oval shapes of three varieties of lettuce. • Creating multiple sizes of solid spheres in the snow while building a snowman. • Planting sunflower seeds in the Greenhouse. They leveled dirt, made holes for the seeds and planted three seeds in a triangular shape. (Filling a pot with dirt is also volume.) • Using cubed, rectangular, cylindrical and triangular-shaped pieces of wood to build a bridge in the Messy Materials Area. • Identifying geometric shapes made by snow on the Climbing/Crawling structure (i.e., triangles, squares, rectangles and lines). • Drawing geometric shapes that represented marshmallows, chocolate, graham crackers and angled skewers (squares, rectangles, circles and lines). • Sketching round acorns that they had collected in the Nature Explore Classroom.

- Creating a large circular "chickadee nest" in the Sand Area that they could fit inside (defining the perimeter using a variety of circular pans and containers that also served as chickadee food).
- Counting three points of a triangular-shaped end of a stick.
- Conducting "interviews" with peers about their preferences and recording each child's response inside geometric shapes the child had drawn on his paper (also one-to-one correspondence–one shape per child).
- Observing sunlight reflecting through a magnifying glass on a table, noticing a "circle" and a "diamond" shape.

Table # 6: Sampling of One-to-One Correspondence, Number and Counting Skills

Child(ren) used one-to-one correspondence, number and counting skills when they:

- Counted six bees that were sitting on six flowers.
- Counted three packages of seeds sitting in the Greenhouse. They sketched the three packages on paper and wrote the names of three kinds of flowers.
- Counted twenty-five plants in the Greenhouse, touching each one as they counted.
- Described four stages of a butterfly's lifecycle made with play-dough. After multiple experiences with butterflies outdoors, she created word labels and placed each by the appropriate stage.
- Counted short and tall cylinder blocks placed in a line. One boy touched each one as he counted.
- Counted twenty flower blossoms in the garden.
- Took orders for three flavors of pretend snow cones, indicating the number of flavors in her response to an order using the phrase, "One triple cone coming up."
- Counted the number of acorns they collected and divided them so each boy could keep one.
- Counted twenty-five rosemary plants in the Greenhouse.
- Counted the number of children who were prepared to play outdoors in the snow by creating a chart. They compared the number of names in both columns and concluded

that five children had proper clothing to play in the snow (three did not).

- Identified "three flower families," while looking for signs of spring in the Nature Explore Classroom and creating a map of all of the signs of spring.

- Counted the number of letters in the word robin. A child wrote the word, then added the number five to represent the number of letters in the word robin.

- Counted the number of tree cookies and wooden blocks while walking on her path path. The little girl who created the path sketched it, including the number of wood pieces on it. Then she checked her work by comparing the number of wood pieces on the path to the number in her drawing.

- Counted the number of caterpillars brought indoors and placed in an aquarium for observation.

- Counted one ladybug on a green bean plant, a second ladybug, then multiple ladybug eggs.

- Counted five birds in the sky, then drew five birds on paper.

- Counted three holes (saying "one," "one" and "one"), using a stick and touching each hole as he counted.

- Counted ten bricks they arranged in a circle to represent a "fire ring."

- Wrote the numbers 1 through 29 with chalk on 29 large chunks of wood, lined the pieces of wood up consecutively for an obstacle course, and counted each piece as they stepped on them. When documenting this experience, one child then wrote the numbers 1 – 29 on a piece of paper.

- Counted 11 keys on the akimbira.

- Lined up and counted 63 seeds from hyacinth pods they had opened.

- Counted how many kinds of nature items (i.e., categories) they had collected. They placed the categories of items in a line and wrote the numbers one through eight by each set of items.

- Counted 105 roly polys in the worm bin, breaking an earlier record of 25 (a challenge given by the teacher to find more than they had found earlier that morning).

- Counted the number of squirrels' nests in trees on a squirrel counting walk.

- Counted 100 kernels of corn (then told the teacher he needed 100 pots to plant the 100 kernels).

- Counted 13 “runners” who wanted to be in a race, and counted 21 total people (the remaining 8 were spectators).
- Counted 32 paw paw leaves collected in the Nature Explore Classroom (teacher prompted at the number 30 when the child got stuck).
- Collected “pizza” orders (at their pretend-play pizza restaurant), drawing circles to represent each child’s pizza order and placing each child’s initial inside his/her circle, then counted the total number of pizzas ordered (continuing to add to the order as additional children ordered).
- Counted each beat on a slap drum (one through eight), drumming in a rhythmic pattern, repeating the count for five minutes.
- Kept score in a “hockey game” (child vs. teacher), by recording tally marks on the cement with chalk, then counting the total number of points for each player.
- Counted snow chunks collected to place inside a piece of gutter that was angled like a ramp (child relied on his fingers inside his mittens to count). Child intentionally selected small chunks that would fit inside the gutter.

Table # 7: Sampling of Skills Experiencing/Using Whole-Part Thinking

Child(ren) experienced and/or used whole-part thinking when they:
• Observed a tree and sketched a part of the tree (one branch). (Also large and small scale). • Drew live bunnies from visual memory and depicted three whole bodies with multiple body parts in different positions. • Observed bumblebees and sketched the bees, including whole bodies and individual body parts. • Collected and used whole acorns and parts of acorns. • Ground corn in the Greenhouse (used whole cobs, kernels to create ground corn).

- Constructed a whole tree out of parts of a tree (sticks and leaves), representing the trunk, branches and leaves.
- Sketched a tree, depicting a symmetrical ‘robin tree’ with all the parts including dirt ball, trunk, branches and leaves.
- Cut baby carrots into small pieces to feed to the worms (also ate the carrots and counted the number they ate).
- Selected large pieces of snow and ice and repeatedly broke them into smaller pieces.
- Dead-headed flowers and collected seeds in the garden (plant, flowers, seeds).
- Picked and opened hyacinth bean pods and removed the seeds (also sorted and lined up the seeds).
- Picked leaves from the nasturtium and zinnia plants (to taste) and compared the taste based on which plant they came from.
- Picked apart milkweed pods so all children had a handful of “magic seeds” and counted the number of seeds in their hands.
- Created a rhythm by tapping on a stump, a whole tree, and pieces of a tree cookie.
- Opened whole locust seed pods, removed individual seeds and divided them into two piles.

Table # 8: Sampling of Children’s Use of Computational Skills

Child(ren) used computation skills when they:
• Added the length of a ruler (12”) and five fingers (representing five more inches) to determine that the depth of a hole they had dug was 17”.

• Used bird feathers for addition, beginning with two feathers they had found and collected additional feathers until they said they had a total of five feathers. • Added the number of acorns a child was holding, beginning with six, then added more until there was a total of nine. • Tapped on nine pieces of wood (broken tree cookies, wood chunks and one stump) in a rhythmic pattern, added another piece to make 10, then stated, "now we'll have to go to eleven." • Counted 12 pumpkins in a pile, then added two more and announced there were 14. • Counted nine "pizzas" children had ordered, and when an additional pizza was ordered changed the order to 10.

Table # 9: Sampling of Children's Use of/Experiences with Fractions

Child(ren) used/experiences fractions when they:
• Intentionally selected and worked with acorn halves (filling them with snow). • Worked with tree cookies that were whole or broken into halves and thirds. • Worked with whole and halves of beans. • Filled an empty bucket half-full with corn kernels, then poured the corn into a larger metal bowl, then poured that quantity into another bucket that was already three-fourths full of sand, causing the bucket to overflow with the excess corn. • Speculated that a nature item they found when digging was a "dinosaur bone" that "could be half the size of the Climbing/Crawling structure."

Table # 10: Sampling of the Skill of Making Comparisons

Child(ren) used the skill of making comparisons when they:
• Compared a small stick to the size of a finger. • Compared a hole full of water to the size of his foot. • Compared shapes of animal tracks on the Nature Explore Classroom to photographs of animal tracks in a book (the object was to identify the animal by its tracks). • Compared the length of icicles to see who had the longest icicle. • Compared the size of a handprint in an organic brick wall to the size of her hand. • Compared three birds observed outdoors to three types of birds on a bird poster indoors. • Compared big birds to small birds while closely observing them through a window. • Compared bugs by noting their properties. They noticed the bugs were the same color but different shapes. • Compared the length of eight corncobs (also counting and classification) and selected the two longest ones. • Compared the size of snowballs they had made (with each other's snowballs) and compared the distance they could throw their snowballs. • Compared the depth of two holes dug side-by-side (and measured with a ruler). • Compared the size of birds, and identified them as "huge" and "huger." • Compared the size of plants in the Greenhouse (specifically a new plant with an already established plant). • Compared the size of long sticks by extending them up into a tree to see how far they

could reach.

- Compared the size of a large leaf to a child's head.
- Compared the size of the akimbira keys with the sounds they made.
- Compared the number of seeds children had in their hands (using gross terms to describe quantity – "a lot" and "a little").
- Compared the length of Dragon Tongue beans (against peers' beans), using a ruler to observe length.

Table # 11: Sampling of Children's use of Matching Skills

Child(ren) used matching skills when they:

- Matched photo/word label markers found in the Greenhouse to the plants in the Nature Explore Classroom.
- Matched the colors of paint samples to the colors of nature in the garden.
- Matched rooted coleus found in the Greenhouse to the mature coleus plant in the Nature Explore Classroom.
- Matched photographs of a variety of lettuces found on seed packets with lettuces that had been planted.
- Matched flowers by type when looking for flowers to taste.

Table # 12: Sampling of Children's use of Measurement Skills

Child(ren) used measurement skills when they measured:
• The depth and width of holes they dug (using rulers, sticks and their hands as units of measure). • The depth and length of the water in the water feature, using a 6' pole as a measuring tool. • How many feet high a plant in the prairie garden was (using a hand as a measuring tool, moving it vertically up the plant as the child counted to eight). • The height of the child's body with a stick, then selected a longer stick to measure the teacher's height (estimated length and selected a stick that was exactly the teacher's height) • The length of several sticks, trying to find two the same length to use as ski poles (examining several sticks against the original stick and discarding those that did not match the length). • The length of Dragon Tongue beans with rulers (wrote the numeral five to represent five inches). • Equal distances from both sides of a center point of a large, rectangular shaped "hockey rink" that the child had drawn with chalk on cement near the Gathering Area (using his feet as the unit of measurement). • The weight of scissors and rulers using a postal scale (reading the number of ounces on the scale out loud). • The length of eight corncobs (also counting and classifying). • The weight of nature items (pumpkins and gourds) placed on a postal scale, and the weight of their bodies (on a floor scale), watching the needles on the scales move, reading the numbers on the scales and discussing the weight in pounds versus ounces.

Table # 13: Sampling of Children's Use of/Experiences with Scale Relationships

Children experienced scale relationships when they:
• Used six-foot long poles to create a 100% scale structure they could fit inside. Then sketched the large-scale 3-dimensional structure on paper creating a smaller scale 2-dimensional drawing of it.

• Placed six-foot poles on the ground to create large-scale letters. Then they sketched the letters they created on paper in small scale. • Built a small-scale wall using mini bricks while observing an artist building a large-scale organic brick wall as a boundary for the Messy Materials Area. • Created a small-scale, eight-inch model of the earth out of snow. • Moved wood chips with their whole bodies to create negative spaces in the Messy Materials Area. The negative spaces represented the first letters of the children's names. • Wrote their names in the dirt with a six-foot long pole,(large-scale 3-dimensional) and then sketched their work on paper (small-scale 2-dimensional).

Table # 14: Sampling of Children's Use of Sequencing and Seriation Skills

Child(ren) used the skill of sequencing/seriation when they:
• Described the steps a bird used to create a nest. • Experienced the process of planting, from planting seeds to transplanting seedlings in a garden. • Described the sequence of their snow cone making process. They described finding the appropriate materials to put snow in acorns, coloring them and putting them in a box for snow cone sellers. • Represented the lifecycle of a butterfly out of clay, then described the stages of metamorphosis to the teacher. • Described the steps of their work with rakes and leaves: 1. Rake leaves, 2. Fill the buckets, 3. Dump leaves out to create a pile, 4. Jump into the pile.

- Created a 2-dimensional sketch of the growth of a flower from smallest to biggest.
- Arranged paw paw leaves by size, from smallest to largest (also compared sizes).
- Used the words “baby.” “mommy,” “daddy” and “grandpa” to depict the sizes of trees on the Nature Explore Classroom (series of small to large trees).
- Created and described a sequenced order of activities when making “candy” to sell to their peers (first fill this bowl, then this bowl, then this bowl…).
- Created small, medium and large snowballs to combine to make a snowman.

Table # 15: Sampling of Children’s Experiences with Volume

Child(ren) experimented with the mathematical concept of volume when they:

- Placed pinecones inside notched openings on an organic brick wall.
- Removed soil from a worm bin in the Greenhouse and filled flowerpots.
- Filled acorn halves with snow to make “snow cones” while packing and leveling each one.
- Filled a bucket with acorns.
- Filled an open space in a small brick cave with tree cookies.
- Filled a bucket with leaves to create a pile on the ground to jump into.
- Filled buckets, metal bowls, cake and cupcake tins with sand.
- Dug a “really deep” hole and watched it fill with water.
- Used a large bucket full of water to fill child-sized watering cans then dumped the

water on plants, (the volume of both sizes of buckets changed).

- Dug a hole with a shovel in the Dirt-Digging Area, experiencing a change in volume while trying to keep loose dirt from falling back into the hole.
- Placed their bodies inside a large hole they dug to see if they could fit inside.
- Filled a large metal bowl with scoops of sand, then added water.
- Placed water, mud and nature items in a clear, plastic container and watched the water level rise as they added materials.
- Filled multiple containers with sand to make “witches' cupcakes.”
- Filled a tea kettle with wet sand that represented soil, then added more sand to represent water to water seeds (i.e., corn kernels) (also experienced the changing weight and when the kettle was full one child stated that it was “really heavy”).

Table # 16: Sampling of Children’s Use of Time Concepts

Child(ren) used/developed time concepts when they:

- Referred to a bird’s nest as “old” and speculated that it took the bird “about five days” to create it.
- Dead-headed flowers and discussed how they would save the seeds to plant “later.”
- Created a building design proposal for a house in the Sand Area and included a representational clock on it.
- Recalled their pretend play and relayed it to the teacher saying it was their “third day” engaging in the same activity.
- Observed a bell tower from the ground. The chiming initially scared the child. He told

his teacher, “When I get older I’ll be able to see them better” (the bells).

- Planted seeds and decided they would check “in a week” to observe the growth over time.
- Estimated the length of time it would take (in seconds) to run around a race track they created.
- Speculated that they had observed a particular pigeon “the year before.”
- Speculated that a piece of driftwood was the “tummy of a “rhinoceros” and was “made a thousand million weeks ago,” and that another piece of driftwood was a “skull from the 1948s.”
- Used a stopwatch with a beeper to begin and end races.
- Stated that “witches' cupcakes” need to bake for “100 pumpkin minutes.”

Prior to the focus group interviews, we asked teachers to bring examples of Nature Notes that reflected children’s math learning. We also asked them to briefly record their thoughts about children’s math learning in the outdoor classroom. We did not specifically ask teachers to list math skills they had observed in the outdoor classroom, however, most did. Table 17 identifies the skills teachers identified most frequently on their interview protocol.

Table # 17: Math Skills Most Frequently Identified by Teachers *

Math Skills	**Number of Teachers who Identified Skill (n = 14)**
Volume	12
Number	11
Counting	9
Estimation	8
Patterns/Repeated Patterns	6
Geometric Shapes	5
Size	5
Symmetry	5
Classification	5

* Note: These skills were identified in writing by at least one-third of the preschool teachers. Eighteen preschool teachers participated in the interview, however only 14 submitted written documentation. This table only represents the skills identified by those 14 teachers. Teachers identified several other skills on their notes, including: making/learning about lines (4), length (4), levels/height (3), perimeter (3), one-to-one correspondence (3), distance (3), seriation/sequencing (3), making equal (3), fractions (3), area (2), time concepts (2), weight (2), width (1), matching (1), division (1), speed (fast/slow) (1), whole-part relationships (1), and depth (1).

Math Vocabulary Children Were Using in their Play in the Nature Explore Classroom

The second theme describes the rich math vocabulary young children were using and learning in the context of their play in the outdoor classroom. Examining children's vocabulary provides insight into their understanding of basic math concepts and illustrates how they experiment with math vocabulary as they describe their world and try to make sense of their observations and experiences. In the following stories we highlight (in bold font) the basic math vocabulary teachers recorded (in quotation marks) on their observation protocols. At the end of the section we provide a more detailed description of children's math vocabulary, categorized by skill.

"Because I know math!" (Grinding Corn)

Four children (four and five years old) worked in the Greenhouse, grinding corn using a manual coffee grinder. They began with ears of Indian corn, took the corn off the ears and put it into the grinder. When they finished, children recorded their experiences on paper, using colored pencils, then shared their experiences with their teacher and peers. The teacher described this as an opportunity for peer teaching. Louisa explained her drawing to her friends: “***First***, the corn starts out like this (on the cob). ***Then*** you put it in the corn grinder. And ***then*** it turns into ***smaller*** pieces of corn which ***equals tiny*** pieces of corn.” Ms. Murdoch asked Louisa, “How do you know the word equal?” Louisa confidently replied, “Because I know math!” “How?” Ms. Murdoch asked. “My brother teaches me. He’s in the second grade. ***1 + 1 = 2, 3 + 3 = 6, 4 + 4 = 8, 20 + 20 = 40***” (she shared more addition facts but the teacher did not record them all) (Holly Murdoch, February 24, 2009).

Scarves for sale!

Garrett (four) was selling large pieces of fabric and colored scarves in the Nature Explore Classroom. He came up to Mrs. Tichota and asked, “Want to buy ***one***? They cost ***$6***.” As she gave Garrett her dollars (woodchips) he counted, “***1, 2, 3, 4, 5, 6***.” He then said, “I have ***three*** (scarves) left – ***orange, blue and green***.” Garret sold Mrs. Tichota one more scarf. He said, “Now I have ***two***.” He went to the Music and Movement Area where he got another scarf. He announced, “Now I brought ***another one*** – I have ***three***.” Garrett retrieved another one, “***Another green one***.” Mrs. Tichota said, “I wonder how many you have now?” Garrett counted, “***1, 2, 3, 4***” and asked, “Want to buy ***another one***? ***Two dollars***.” She said “Okay” and gave Garrett one dollar (woodchip). Garrett said, “It’s ***$2***,” and Mrs. Tichota asked, “How many more dollars do I need?” Without hesitation, Garrett replied, “**One**.” (Kathy Tichota, December 12, 2012).

Measuring Corncobs

Cade went throughout the Nature Explore Classroom collecting corncobs. He dropped a load at his teacher’s feet on the concrete in front of the Greenhouse. Cade said, “Look at ***all*** those corncobs!” and Mrs. Tichota replied, “Wow, I wonder how many there are?” Standing over the pile, pointing as he counted, he said aloud: "***1, 2, 3, 4, 5, 6, 7, 8 – eight***." Mrs. Tichota asked, "Which one do you think is the longest?" Cade picked up the two longest corncobs (by visually examining them) and held them side-by-side. "What are you doing?" Mrs. Tichota asked. Cade replied, "I'm ***measuring***. They're ***both big*** – ***this one's the biggest***," and held out the longest corncob (Kathy Tichota, January 14, 2013).

"We're going to make lots of money! Who likes candy?"

Four-year old James and three-year old Isaac were playing in the Sand Area. James had a plan to make and sell candy, and told his teacher: "I've got these ***three*** bowls for my plan." He explained the process, "***First, we fill up this one***." He filled that bowl until it was completely full. "Now," he said, "we ***flatten this*** (to level)." He used a rolling pin and flattened the top of the sand in the bowl. By then, Isaac became excited about James' plan and joined in. Jack said, "We are ***all*** diggers, aren't we?" Isaac, who was using a funnel to dig, enthusiastically replied, "Yeah!" James gave his younger friend a bit of instruction: "No, don't use that ***one***…it has a hole in it" (referring to the funnel). Jack announced: "This is the candy that we're gonna put in ***all three bowls*** when we're done!" As he talked he filled a measuring cup, dumped that into a cylindrical container, and then added the sand to the bowl. Isaac made sure he understood what they were doing, "We're mixin' it up, then we ***fill that one up***–right, James?" Isaac continued to use the funnel to dig, and added the sand in his funnel to another small cylindrical container, then added it to the second bowl. As Isaac added sand to the bowl he said, "Pepper, Pepper, Pepper!" "No," James countered, "not pepper – CANDY!" "Hey, Isaac," he added, "We're going to make ***lots*** of money! **Everyone** likes Ice cream…who likes candy, who likes candy?" he chanted (Katie Logan, December 6, 2012).

We are Chickadees

When the children in Mrs. Tichota's group entered the NEC, they went to a section of colored bricks that they called their "chickadee nest." They often pretended when making their daily plan outdoors that Mrs. Tichota was the mother chickadee and the children were her "little chickadees." On this day, Chelsea and Patty planned to play in the Sand Area. They used shovels and a hoe to dig a large hole in the middle of the Sand Area. They announced, "It's a chickadee nest." Chelsea sat in the nest. Patty said, "Get diggin'," and Chelsea replied, "I'm a chickadee. I sit in the nest." Patty, using the hoe to make the hole (nest) bigger, stated, "This makes the nest." Chelsea said, "I'm gonna be the ***dad***…you be the ***mom***." As she picked up two corncobs, Chelsea said, "***Two*** chickadee foods." Chelsea put the corncobs in the nest and said to Patty, "You go out and find ***some*** food." Patty reminded Chelsea: "Remember, I'm the mom and you're the dad." Chelsea lowered her voice to sound like a dad and chirped, "chickadee-dee-dee-dee-dee. Remember, mom sits on the nest." Brandon and Anna, who had been playing nearby in the Sand Area announced, "We're making chickadee food." Chelsea said, "I'll sit on the eggs." "No, I will!" Patty exclaimed. Patty went and gathered buckets of sand. She said, "This is chickadee food. We have to ***wait for the eggs to hatch***." Patty told Anna, "You can be a ***baby*** chickadee." Anna used a small shovel to fill a large metal bowl with sand and said, "***More*** chickadee food." The children had started to place sand-filled containers (buckets, bowls) around the outside of the chickadee nest in the shape of a circle… Patty carried a large stone and placed it by the nest. She said, "It's for the grown-ups to sit on" (thinking about size relationships). Together Ally and Harper delivered a large bowl filled with sand. They said, "***More*** chickadee food." Patty said to Harper, "My name is Chickadee. Can you say that?" As the children filled more and more buckets and bowls with sand they chanted, "***More*** chickadee food, ***more*** chickadee food, ***more*** chickadee food…" Patty said, "Harper is the ***five-year old bird***." Harper chirped, "chickadee-dee-dee." Chelsea said to Patty, "Mommy chickadee, I'm gonna fly for some ***more*** food." Chelsea spread her arms and flew out of the Sand Area. Anna, after filling many containers with sand, said, "I'm ready to play." Patty began to explain to Anna her role as a baby chickadee. Chelsea, who had flown back into the Sand Area, said, "I'll show her." Chelsea and Patty both demonstrated to Anna how to sit on the nest. Anna asked, "Can I be hatched ***right now*** and sit in the nest?" and lay down in the nest. Harper asked, "Is this the nest?" Chelsea replied, "Yes, you're the ***baby***. You can only say 'chick'." Unfortunately, it was time to clean up. The children asked if they could leave the nest and chickadee food out. Mrs. Tichota told them that would be a good idea because maybe another group of children might want to be chickadees - (Kathy Tichota, December 7, 2012).

Guess What I Found? (Feathers)

Lisa (four-years old) had been in the Sand Area playing with sand and water when she noticed Hannah and Mrs. Tichota sitting on a bench. Lisa walked over and said, "Guess what I found?" She held up two black bird feathers that were about six inches long. She offered the feathers to Hannah but Hannah said, "No, I don't want them, they're wet." Lisa wandered off and soon returned. She said, "Look – ***three***." She had found another feather. She left again and a couple of minutes later found Mrs. Tichota, who was then sitting by the climbing/crawling structure. She was holding three feathers in one hand and one feather in her other hand. She said, "***Three and one makes four*** feathers!" Lisa found one more feather and announced: "***Now I have five***" (Kathy Tichota, April 29, 2013).

Collecting Acorns

While walking the paths through the Nature Explore Classroom, Caleb (five-years old) and Jay (four-years old) picked up acorns and added them to a bucket. Jay said to Mrs. White, “We have a bucket ***half-way full***.” Caleb said, “It’s ***more than half way***!” Mrs. White drew a bucket on her documentation form and Caleb drew a line across it to show how full their bucket was. Mrs. White asked them, “What makes you want to collect acorns?” Jay replied, “We’re making a home for them. Maybe we should hide them around the school and see if a squirrel finds them. I’m hiding ***one*** over here!” Jay walked to the fence and placed an acorn behind a rock. Caleb said, “We’re hiding them from the squirrels – we’re playing hide and seek with the squirrels.” Caleb said, “Mrs. White, I have ***six*** in my hand,” and showed her his left hand. “And look at Jay!” Jay placed all the acorns he had picked up into one hand and said, “I have ***six*** too!” They put acorns in two buckets and one pan, and eventually Caleb said, “Look ***how much*** we have!” They went to the block area to use the table to “make pie.” Jay asked Mrs. White, “Guess what the secret ingredients are?” and opened his hand and showed her two acorns (Joyce White, August 29, 2012).

As an extension to this activity, six days later Caleb again collected acorns. “Mrs.White,” he said, “I have **nine** in my hands right **now**.” He opened his hands. He had sorted the acorns by specific properties. In his left hand, he had five dark colored, larger acorns. In his right hand he had four smaller green acorns with lids. The teacher traced Caleb’s hands on a documentation form and asked him to add the acorns. He drew five acorns in one hand, without lids, and four acorns in the other hand, with lids” -(Joyce White, September 4, 2012).

Comparing Snowballs!

Mrs. White's class was headed outdoors on a late January day. The group had made their plan and their goal was to find one thing that had changed in the outdoor classroom that day, compared to the day before. The previous day had been warm (for January) and the snow and ice had melted. The children discovered that the melted snow from the day before had turned to ice, and any remaining snow was packed down like ice. Greg, David and Carl (all five) made snowballs out of chunks of snow. Carl told David, "My snow ball looks ***the same size*** as yours." David replied to the teacher instead of to Carl: "Yeah, but Carl's snowball is ***a little bigger than mine***." But David kept adding chunks of snow to his snowball, and soon announced, "My snow ball is ***way bigger now***!" Carl threw his snowball all the way from the Greenhouse door to the open space. "Look it," he exclaimed, "Look it ***how far***!" At this point, all three boys began throwing snowballs and comparing whose snowball went the farthest distance. Greg formed a snowball and began rolling it on the ground, down on his hands and knees. He said, "I'm going to make this ***bigger***" and showed his friends and teacher how the snowball increased in size as he rolled in through the snow (Joyce White, January 28, 2011).

Tables 18 through 24 provide a sampling of the math vocabulary children used in their interactions with teachers and peers during their play. We only included words and phrases that teacher/co-researchers specifically recorded in quotation marks, to ensure accuracy. The vocabulary tables are categorized by math skills, including:

- Classification (Table 18)
- Computation (Table 19)
- Measurement (Table 20)
- Number/Counting (Table 21)

• Quantity (Table 22)

• Size Relationships (Table 23)

• Time Concepts (Table 24)

Table # 18: Vocabulary That Illustrates the Skill of Classification – Identifying and Sorting Objects by Type and Properties

Gender/Age	Child's Words	Contextual Reference
Nine children Ages three to five-years-old	"They're different sizes." "They're hard." "They're different colors."	Children described the attributes of a pile of beans they had sorted by type.
Female Three-years-old	"A caterpillar. A ladybug."	Child brought insects indoors, placed them in an aquarium for observation and classified them by type.
Female Four-years-old	"Oval, it looks like an oval kind of shape."	Child classified the shape of a lettuce leaf she was observing.
Male Four-years-old	"I can't remember what this is, but it's too big for a roly poly."	Child tried to identify a type of insect he was holding in his hand by eliminating possibilities.
Female Four-years-old	"Some have flowers on them. These are crab apples. I have a big apple tree in my backyard."	Child identified a specific type of tree while observing and sketching the tree in the Nature Explore Classroom and classified a tree in her backyard by size and type.
Male Three-years-old	"It's a half-moon."	Child answered his teacher's question about the kind of tool he created (a curved piece of driftwood he was using as a scoop in the snow) by classifying its shape.
Male Six-years-old Male Six-years-old Male Six-years-old	Child # 1: "A baby." Child # 2: "A junior." Child # 3: "It's a two-year-old. There's so many broken ones."	Three children sorted acorns they collected into buckets. They created their own classification system based on size.

Table # 19: Vocabulary That Illustrates Computation Skills

Gender/Age	Child's Words	Contextual Reference
Male Four-years-old	"It's bigger than the ruler. I have seventeen now."	With the help of his teacher, child measured depth of hole he had dug and counted twelve inches. He marked the spot and counted five more inches. He used his five fingers to add twelve plus five.
Female Four-years-old	"Look, three. Three and one makes four feathers. Now I have five."	Child found bird feathers and added them as she showed the feathers to her teacher.
Male Five-years-old	"I have nine in my hand now."	Child's teacher traced his hands and he drew five acorns he had collected in one hand and four in the other. Child added his acorns.
Female Three-years-old	"Now we'll have to go to eleven."	Child counted to ten as she tapped on tree cookies with a stick. She added one more tree cookie.
Male Four-years-old	"I found two, now we have fourteen. There were twelve and two more equal fourteen."	Child mentally calculated the total number of mini-pumpkins sitting on a stump.
Male Four-years-old	"I have three left – orange, blue and green. Now I have two. Now I brought another one – I have three. Another green one…1, 2, 3, 4. Wanna buy one? Two dollars. It's two dollars. One."	Child subtracted and added as he sold fabric and scarves to his teacher. Teacher gave him one dollar for a scarf. He reminded her it was two dollars and said she needed one more dollar.

Table # 20: Vocabulary that Illustrates the Skill of Measurement

Gender/Age	Child's Words	Contextual Reference
Male Five-years-old	"Awesome! Mine goes the farthest."	A child compared the distance his car traveled down an outdoor ramp to the distance his friend's car traveled.

Male Five-years-old Male Five-years-old	Child #1:“Look how big this stick is. It’s five or fifty feet.” Child #2:“It’s big enough to touch the tall tree leaves.”	Child #1 described the length of the long pole he was manipulating in the Messy Materials Area. Child #2 responded to his friend about the length of the long pole his friend was carrying.
Male Five-years-old	“It says one” (cup).	Child read the label on a measuring cup as he poured melted ice into it.
Female Five-years-old Female Five-years-old	Child #1: “A California condor wingspan is nine to ten feet long.” Child #2:“You know my apartment building is nine feet tall, too!”	Child #1 compared the wingspan of a California condor to the length of a long pole she was carrying. Child #2 replied to her friend who had described the wingspan of a California condor.
Male Five-years-old	“You went higher than me.”	Two children were observing birds. They assumed the role of those birds as they watched the birds fly into a tree. One child told his friend that the friend flew higher.
Male Five-years-old	“I’ll just reach it with a stick.”	Child informally measured the distance to a branch he was trying to touch and selected a long stick that would reach that branch.
Male Five-years-old	“Fourteen feet long.”	Child tried to “fix this wall” by adding a long pole to a “house” he had helped build. He tried adding poles that were too short and calculated he needed one that was “fourteen feet long.”
Male Six-years-old	“Sixty pounds… I’m making mine deeper than his…Look, it goes up to my knees.”	Child responded to teacher’s question about weight of sand. He jumped in a hole he had dug and used his body as a standard of measure.
Female Three-years-old	“Yes, that’s what we need to measure.”	Child replied to teacher’s suggestion to get a ruler to measure the depth of the hole she had dug.
Male Four-years-old	“1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12…it’s 12 inches.”	Child counted the numbers on a ruler as he measured the depth of a hole his friend had dug.
Male Five-years-old	“See how deep it is! I dug that deep – it goes down…it goes to there.”	Child used a stick to dig three holes in the snow and used the stick to measure deepest hole.

Female Four-years-old	"Wow, look how long this is! One hundred, yeah, that would be really long."	Child used long stick to reach up in a tree and replied to teacher's question about the length of the stick.
Female Three-years-old	"Look, how long mine is – really long."	Child placed a stick on the ground and examined its length.
Male Four-years-old	"This is longer." (Child pointed to water feature.)	Child responded to teacher's question about which was longer – the stick the child had or the water feature.
Male Four-years-old	"See it comes up to here now. When I add more stuff, the water goes up to here."	Child used his flat hand to measure change in water level as he added more nature items to clear container of water.
Male Four-years-old	"This is how long the bean is."	Child drew a line on paper along the side of the bean he had harvested.
Male Five-years-old	"It's five. Can you write five?"	Child traced dragon tongue bean he had harvested and measured it with a ruler.
Male Five-years-old	"It went down the slide and it went one hundred miles per hour."	Child described speed of the long log he pushed down the slide (part of his "log experiment").
Male	"This is how long the bean is."	Child drew line along the side of the bean he had harvested.
Male Five-years-old	"Eight feet, I'd say."	Child responded to teacher's question about the length of "dinosaur bone" he'd discovered.
Male	"I made a very deep hole."	Child described hole he had dug and measured with a ruler.
Male Four-years-old	"Four"	Child placed two pairs of scissors on scale and read where indicator pointed.
Male Four-years-old	"I'm measuring. They're both big. This one's the biggest."	Child measured two corncobs.
Female Five-years-old Female Four-years-old Male Five-years-old Male Four-years old	Child #1: "Thirty-eight." Child #2: "Zero four…one, one, four. Child #3: "I weigh thirty-two. Look, Ms. Miller, how much this weighs. I weigh all the way to the bottom." Child #4: "I know what this is – you put something on it	Four children experienced weighing themselves, pumpkins, and gourds on two different scales, a postal scale and a floor scale.

	and it shows how much weight it is. It's fifteen, eleven, this is three. I know this one is bigger."	
Female Five-years-old	"It's as tall as me. If I tried this stick next to you, it would be so small."	Child measured her height with a stick then compared stick to teacher's height.
Female Five-years-old	"Let's see how big you are. It's as tall as you.	Child selected a stick the same height as her teacher.

Table # 21: Vocabulary That Illustrates the use of Number/Counting Skills

Gender/Age	Child's Words	Contextual Reference
Female Three-years-old	"One green and speckled frog…One jumped into the pool…Now there are no green speckled frogs."	Child sang as she sat on a log and used her body to act out the words to the song.
Female Six-years-old	"A clock on the tower…two windows."	Child verbally described to her teacher a design plan she had drawn for an outdoor building project.
Male Four-years-old	"Actually, I only need two. I was trying to reach that branch."	Child subtracted two tree cookies from a stack of four when he realized he only needed two tree cookies to stand on to reach a branch on a tree.
Male Three-years-old	"My family, there are three flower families."	Child described a map of the Nature Explore Classroom he had drawn so his teacher could label it. He related the number of flowers he drew to the number of people in his family.
Male Six-years-old	"We could each keep one."	After filling a bucket with acorns, a child suggested that he and his four friends could each keep an acorn.
Male Five-years-old	"There's two big roly polys and one cricket."	Child described two types of creatures he found under a log.
Female Four-years-old	"How many caterpillars are in here? I see only two."	Child noted the number of caterpillars she observed in an aquarium.
Male Five-years-old	"Here's a little bug…two little bugs."	While walking through the garden, a child observed two bugs.
Male Five-years-old	"Did you see my holes? I dug three holes."	Child identified the number of holes he had dug as he built a "house."

Male Four-years-old	"I had two beans, and Mrs. Murdoch's out of beans."	Child referred to the number of green beans he had sampled.
Male Four-years-old Male Five-years-old Female Four-years-old	"1, 2, 3, 4, 5, 6, 7, 8, (up to)…25."	Assisted by their teacher when needed, three children counted the number of rosemary plants in the Greenhouse.

Male Five-years-old	"1, 2, 3, 4, 5."	After observing birds outdoors, child sketched a bird and wrote the word "ROBIN." He counted the number of letters in the word.
Male Four-years-old	"Math problems, 1, 2, 3, 4."	Child played with mini-pumpkins outdoors and indoors. He divided the pumpkins into two rows of four and counted eight. Then he wrote "math problems" (in his inventive writing style) and counted as he wrote the numerals on paper.
Female Five-years-old	"1, 2, 3, 4, 5, 6."	Child counted six bumblebees in the garden and drew each bee on top of one flower.

Male Four-years-old	"I saw five birds. I have five X's."	Child was a bird watcher and made X's on his paper to represent the birds he saw.
Female	"Ms. Miller, can I get two shovels."	Child asked permission to get shovels to dig.
Female Three-years-old	"Look, Ms. Murdoch, 1, 2, 3."	Child counted corners on a triangular shaped stick.
Female Four-years-old Female Five-years-old	Child #1: "We went the loops eleven times." Child #2: "I passed down the stairs and then we went back up – we did this two times."	Children experienced being in the rain, documented their experiences by drawing maps, and dictated words to their teacher.
Male Five-years-old	"I have six in my hand."	Child noted the number of acorns he had collected.
Female Four-years-old	"It has three plus three legs on both sides and two antennas."	Child described the parts of an insect she had observed and sketched.
Male Four-years-old	"We are counting the corn and do you know what? We have counted one hundred corns and now we'll have to find one	Child gathered kernels of corn in a large tub and showed them to his teacher.

	hundred pots for the one hundred corns."	
Male Four-years-old	"This is a race for twenty-one people."	Child constructed a running track that could accommodate all the children in his group.
Male Four-years-old	"There's eleven of them. I counted them as I banged them."	Child counted bars on the akimbira as he stuck each one with the mallet.
Male Four-years-old	"The stick is the pole so I can ski. I need one more stick so I can ski."	While standing on two long blocks of wood, child indicated he needed one more stick.
Male Four-years-old	"One cake batter, sugar, some evil frosting, one hundred pumpkins and one evil leaf."	Child provided teacher with his recipe for "witch cupcakes" that he had made in the Sand Area.
Male Four-years-old	"We're here in Colorado seeing fifty to fifty-four horses."	Child provided information about a trip he and his friends were taking.
Male Five-years-old	"10, 9, 7, 8, 9, 10."	Child counted down as his friend who was "faster than a cheetah" took off running.
Male Five-years-old	"1, 2, 3, 4, 5, 6, 7."	Child counted the "magic seeds" (milkweed seeds) he held in his hand.
Female Three-years-old	"1, 2, 3, 4 – 1, 2, 3 – 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Here, you get two sticks. You count to ten."	Child counted musical taps made by tapping on tree stumps, wood chunks, tree cookies, and a tree trunk with a stick. She instructed and modeled what she wanted her teacher to do.
Female Three-years-old	"This is old money. This is new money. I need this money to buy a surfboard…1, 2, 3."	Child referred to wood chips and leaves as money she needed to purchase a surfboard.
Female	"What is the first letter of your name? I don't need your whole name, just the first letter…Mark (and Max) wanted a pizza so we have two M's. 1, 2, 3, 4, 5, 6, 7, 8, 9 – almost 10."	Child recorded first initials of children's names with a circle drawn under each initial as she took pizza orders. She counted the number of pizzas sold.
Female Four-years-old	"1, 2, 3, 4, 5, 6, 7, 8."	Child counted and sang each beat as she drummed.
Female Three-years-old	"I see a squirrel eating some nuts. It's eating three nuts."	Child described activity of a squirrel she observed in an oak tree.

Male Five-years-old	"Hockey one, hockey, two, hockey three…the tally."	Child tapped stick on the ground, then tapped opponent's stick to begin a hockey game. Child created T shape ("tally") used to keep score for the hockey game.
Female Five-years-old	"Two chickadee foods."	Child labeled two corncobs.
Male Four-years-old	"Look at all these corn cobs. 1, 2, 3, 4, 5, 6, 7, 8 – eight."	Child responded to teacher's question about number of corn cobs he'd collected.
Male Four-years-old	"1, 2, 3, 4, 5, 6, 7, 8, 9, 10 – ten."	Child touched bricks with a stick arranged in a circle as he counted them.
Female Three-years-old	"1, 2, 3."	Child counted carrots as she ate them.
Female Four-years-old	"1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13."	Child counted on her fingers the number of carrots she ate.

Table # 22: Vocabulary Used to Describe Quantity

Gender/Age	Child's Words	Contextual Reference
Male Four-years-old	"Look, I got a whole, whole bunch."	Child described a quantity of leaves he collected.
Female Four-years-old	"We need more of that stuff. We need hundreds."	Child referred to the amount of soil and rocks she needed to fill a "hole in the oven" (large hole in a tree stump).
Male Five-years-old	"Let's use all the cookies."	Child indicated the amount of tree cookies he desired to use in the "house" he was making.
Female Five-years-old	"It shreds. Now you need some more, more. Let's fill it up for you."	Child demonstrated the use of a corn grinder to another child.
Male Five-years-old	"I got a few of each except lamb's ear."	Child described the quantity of leaves he had collected from the garden (only collected one lamb's ear).
Female Five-years-old	"I want to put them with the other plants to have more flowers."	Child pulled seed petals off dried zinnia heads and wanted to put them with the already stored seeds.
Male Five-years-old	"Oh, I made a lot of dirt. There, I made a tree. Now I have to make a robin."	Child described to his teacher a sketch he had made of a tree with a large ball of dirt for roots.

Male Four-years-old	"I found some more leaves for our pile."	Child raked leaves to add to an existing pile for children to jump in.
Female Five-years-old	"Let's go get some more!"	Child suggested to her friend that they go gather more kernels of corn to feed to the birds.
Female Six-years-old	"One triple cone coming up. It's raspberry, blueberry, and grape."	A group of five children were using acorn lids as snow cone containers. Child identified three acorn lids that were connected as a "triple cone."
Male Five-years-old	"Awesome! Mine goes the farthest."	A child compared the distance his car traveled down an outdoor ramp to the distance his friend's car traveled.
Male Five-years-old Male Five-years-old	Child #1:"Look how big this stick is. It's five or fifty feet." Child #2:"It's big enough to touch the tall tree leaves."	Child #1 described the length of the long pole he was manipulating in the Messy Materials Area. Child #2 responded to his friend about the length of the long pole his friend was carrying.
Male Five-years-old	"It says one" (cup).	Child read the label on a measuring cup as he poured melted ice into it.
Female Five-years-old Female Five-years-old	Child #1: "A California condor wingspan is nine to ten feet long." Child #2:"You know my apartment building is nine feet tall, too!"	Child #1 compared the wingspan of a California condor to the length of a long pole she was carrying. Child #2 replied to her friend who had described the wingspan of a California condor.
Male Five-years-old	"You went higher than me."	Two children were observing birds. They assumed the role of those birds as they watched the birds fly into a tree. One child told his friend that the friend flew higher.
Male Five-years-old	"I'll just reach it with a stick."	Child informally measured the distance to a branch he was trying to touch and selected a long stick that would reach that branch.
Male Five-years-old	"Fourteen feet long."	Child tried to "fix this wall" by adding a long pole to a "house" he had helped build. He tried adding poles that were too short and calculated he needed one that was "fourteen feet long."

Male Five-years-old	"You see that one hole and that one hole and that one hole."	Child used a stick to dig three holes in the snow.
Male Four-years-old Male Five-years-old	Child #1: "We have a bucket halfway full." Child #2: "It's more than halfway."	Children described the quantity of acorns they had collected in a bucket.
Male Four-years-old	"See, I got all sorts of stuff in there and some water. When I add more stuff, it makes more water."	Child experimented with filling a clear plastic cylinder with nature items and water.
Female Five-years-old	"Tucker just has one, I have a lot. I have a lot, he has a little."	Child compared the quantity of seeds she had to the amount of seeds another child had.
Female Four-years-old	"I'm gonna pretend water and pour on it and grow, grow, grow. I'm gonna fill it all the way to the top. Really heavy – this is very full for a big plant. I have to lift up the pot and pour it in – all of it."	Child pretended sand in a tea kettle was water. She poured the "water" on the kernels of corn she had planted.
Four children Ages three and four	"More chickadee food, more chickadee food, more chickadee food…"	Children chanted as they filled buckets and bowls with sand and placed them around the chickadee nest other children had created.
Male Four-years-old	"I got these three bowls for my plan. First we fill this one, then we fill that one, then we fill this one. This is the candy that we're going to put in all three bowls when we're done. Hey, we're gonna make lots of money!"	Child filled a measuring cup with sand, poured it into a cylindrical container, and then transferred it into a large bowl.
Male Four-years-old	"These are mine and those are yours."	Child divided locust seeds into two piles.

Male Five-years-old	"A hundred of them, maybe a million."	Child estimated how many nature items another child had collected.
Male Four-years-old	"Ninety-six…ninety-six…too many for me to count."	Child responded to teacher's question about how many marigold seeds he thought he had.
Female Four-years-old	"Maybe there is less – nine or thirty."	Child estimated the number of hyacinth seeds her group had

		discovered after another child had estimated thirty-six seeds.

Table # 23: Vocabulary That Described Size Relationships

Gender/Age	Child's Words	Contextual Reference
Male Four-years-old	"I'm gonna make it a big, big pile up to the tree."	Child described the size of his leaf pile he was going to create.
Female Five-years-old	"They (wet beans) are bigger."	Child compared the size of wet beans to dry beans.
Female Three-years-old	"It's big and small."	Child described a wall she had created with two different sizes of cylinder blocks she had lined up in the outdoor Block Building Area.
Male Five-years-old	"I wonder what this little white thing is? It's the same as that one."	Child compared two bugs and classified them as the same.
Male Four-years-old	"Then it turns into smaller pieces of corn which equals tiny pieces of corn."	Child described the size of corn kernels before and after they were put in a grinder.
Female Five-years-old	"I found a handprint. It's smaller."	Child compared the size of her handprint to a handprint she discovered in the brick wall.
Male Five-years-old	"It was one and now it's more. It gets bigger when it's frozen."	Child observed a chunk of ice as it melted in a measuring cup. He refroze it and commented on the difference in size between the liquid and solid.
Female Four-years-old	"It's a dot. I found a big dot."	Child compared a large rock she found buried in the snow to a big dot.
Five Children Ages three and four	"Mine's the biggest."	Children each found one icicle that had dropped from the Greenhouse roof and compared the length.
Male Five-years-old	"The stick was about the same size as my finger."	Child compared the size of a stick in a bird's nest he was examining to the size of his finger.
Male Four-years-old Male Four-years-old	Child #1: "I saw a huge bird." Child #2: "I saw a huger one."	Children compared the size of birds they observed flying in the sky.
Femal	"Let's make a really big	Child described size of hole needed

Four-years-old	hole…the biggest."	to plant peas.
Male Four-years-old	"Mine is not very big. This one is bigger than mine. It's really, really, really tiny."	Child compared size of his Swiss chard plant to another plant.
Male Four-years-old	"I need a big bowl because I'm building a big castle. I think we need a smaller one (brick) too."	Child described the size of bowl he needed for his castle and the size of brick needed for the castle wall.
Male Three-years-old	"Look at this big leaf."	Child commented to teacher as he compared the size of a leaf to the size of his head.
Male Four-years-old	"The bigger ones they are, the lower sounds they make."	Child described pitch of bars on akimbira related to the size of the bars.
Male Five-years-old	"There's some big ones too, like this one. Is this one the biggest? Which one is next biggest? I think that one…then this one. That one is not the biggest."	Responding to a suggestion from his teacher, child arranged paw paw leaves he had collected in order of size.
Male Five-years-old	"This could be half the size of the Climbing/Crawling structure."	Child compared size of the "dinosaur bone" he'd discovered to the size of the Climbing/Crawling structure.
Female Three-years-old Male Four-years-old Male Four-years-old	Child #1: "Oh, that's the mommy tree." Child #2: "That can be daddy." Child #3: "And that can be the grandpa."	Children labeled trees by sizes-shortest to tallest.

Male Three-years-old	"Wow, what a big leaf…there's two big leafs and one little one."	Child compared the sizes of the leaves he had collected.
Female Four-years-old	"It's for the grown-ups to sit on."	Child carried large stone and placed it by "chickadee nest" in the sand.
Male Five-years-old	"It's a small size. Now watch, a medium size."	Child described snowballs he had made.
Male Three-years-old	"This is tricky. When you're on the tallest, then you are tall. When you're on the lowest part, you	Child compared height changes while standing on brick edging.

	would be small."	
Male Five-years-old Male Five-years-old Male Five-years-old	Child #1: "My snowball looks the same size as yours." Child #2: "Yeah, but Connor's snowball is a little bigger than mine. My snowball is way bigger now." Child #3: "I'm going to make it bigger."	Three children compared the sizes of snowballs they made.
Male Three-years-old	"I cut small pieces for the worms."	Child described carrots he had cut to feed to the worms.
Male Five-years-old Female Five-years-old Male Four-years-old	Child #1: "I remember the big tree was hard to move. I couldn't carry it all by myself." Child #2: They were very heavy. But the little ones were not heavy. The little ones we could easily carry." Child #3: "We moved all those trees. The small tree – I could carry it all by myself."	Children recalled the sizes and weights of recycled Christmas trees they had moved.

Table # 24: Vocabulary That Illustrates Thinking About Time Concepts

Gender/Age	Child's Words	Contextual Reference
Male Four-years-old	"This is very fun. I could stay here for a whole hour. What time is it?"	Child was grinding corn in the Greenhouse and used a length of time to emphasize how much he enjoyed it.
Female Five-years-old	"I guess it would take the birds five days to make it."	Child estimated the length of time she thought it would take the birds to build the nest she was examining.
Male Five-years-old	"This is our third day."	Child responded to his teacher's comment about noticing him and his friends engaged in this play scenario before.
Female	"I can tell time with this. By the shadows…It's play time now…We just made lunch	Two girls used long bamboo poles to create a large scale hexagon shape that represented "a clock."

	time. Let's make breakfast time…It's going to be time to go when the shadow is on gate time."	
Female Five-years-old Female Five-years-old	Child #1: "1, 2, 3, 4, 5. It's done." Child #2: "My mom always cooks it for a long time."	The girls used muffin tins to make "cupcakes" in the Sand Area. They pretended the storage cupboard was "an oven." Child #1 estimated the baking time using numbers. The children sampled the (pretend) cupcakes and child #2 put them back in the oven noting the need to bake them longer.
Male Three-years-old	"Look, look, teacher. The plants are growing."	Child observed changes over time in growth of plants he had been watering daily.

<u>Materials That Supported Children's Math Learning in the Nature Explore Classroom</u>

The third theme describes the kinds of materials children used in their play to support their math learning. Many of the materials children collected, classified, counted and compared were natural, loose materials, some naturally found in the environment, others supplied by teachers (e.g., sticks, tree stumps, wood chips, acorns, pine needles, fallen leaves, tree cookies, rocks, cut tree block). Many of the "materials" teachers identified in their Nature Notes were living, growing, organisms (e.g., plants, flowers, trees, insects, birds, and squirrels) that children observed in the environment, and over time. Some of the materials were seasonal such as pumpkins, snow and ice. Other materials teachers noted in their documentation included tools and/or objects provided specifically to support children's skill development (e.g., scales, a thermometer, rulers, shovels, funnels, containers, baking tins, collection bowls, fabric, clipboards and writing materials). In many cases, without these materials (and often, teachers supplying them or setting them up) children would not have had the rich learning experiences teachers described. We share several examples in this section that illustrate a variety of materials children used, then three tables that provide a more comprehensive view of the materials teachers identified in their Nature Notes and the focus group interviews.

Opening Bean Pods

During Materials Exploration time in the Greenhouse children were being "scientists" by cutting open hyacinth bean pods to find the beans inside. Children used plastic knives to cut open the bean pods. They placed the beans in one container and the skin of the beans in another container. Each child added his or her seeds to the community pile. Mrs. V. asked children to take a "scientific guess" on the number of seeds they had accumulated. Robbie guessed, "36." Sara said, "or maybe there is less…9 or 30." Sara and her friend Riley were so engaged that they did not want to stop opening pods when Materials Exploration time was over. So after Music and Movement time, Mrs. V. took the materials and those two children back to the Greenhouse where they continued to use the knives to open the bean pods. At one point Sara said, "I know there is a couple more beans (in a pod she was touching), because there's a couple of bumps." When they were finished opening the pods, they lined up all their beans in front of them on the table in the Greenhouse (Kris Van Laningham, October 1, 2012).

Materials: Beans, Plastic knives, Two containers for sorting

Learning About Scales

Abby pulled out the floor scale in the Greenhouse and stood on it. She read off the numbers – 3 & 8 (38 lbs). Then her friend Jamie stood on the scale. Mrs. Miller asked her: "What are the numbers? How much do you weigh?" Jamie replied, "zero, four" (the scale said 40 pounds). Kai then weighed himself and announced, "I weigh 32." Then Kai switched to the smaller postal scale. He weighed a small pumpkin and exclaimed, "Look, Mrs. Miller, how much this weighs – it weighs all the way to the bottom." Mrs. Miller showed him that the scale said the small pumpkin weighed 1½ pounds. Then Jamie began weighing small items on the postal scale (gourds and small pumpkins). When she told Mrs. Miller the weight, she read the numbers as "1 – 1 – 4" (1¼ lbs.) and "1 -1" (11 ounces). Mrs. Miller explained what the numbers meant, and that the numbers showed how much the items weighed. About that time Harry came into the Greenhouse from outside and saw the postal scale. He said, "I know what this is. You put something on it and it shows how much weight it is." Harry weighed a small pumpkin and said "It's 15" (ounces). He weighed another pumpkin and said, "It's 11" (ounces), then weighed a dried up gourd: "This is 3." Mrs. Miller asked Harry, which one weighed more (the 11 ounce pumpkin or the three ounce dried up gourd). Harry weighed the pumpkin again as he thought about it. "This one" he said, holding up the pumpkin. "I know," he added, "because this one is bigger." (Mrs. Miller noted in her documentation that the pumpkin was not necessarily bigger but they were different shapes, and Harry was correct, the pumpkin weighed more.) (Sherry Miller, January 3, 2011).

Materials: Floor scale and food scale, Mini-pumpkins and a gourd

Bird Watching

Two four-year-old boys went outdoors with their class on a cold winter day. The snow was soft and wet and the teacher encouraged any children without boots to stay on the concrete or work in the Greenhouse. David and Mitchell did not have boots, so Mrs. White suggested they could be "bird watchers." "I need binoculars," David said as he headed to the bench in the Nature Art Area. Mrs. White and Ms. Murdoch had brought two pairs of binoculars outdoors, just in case anyone wanted to use them. David and Mitchell sat on the bench silently, watching the sky when five black birds flew by in the shape of a "V." "Wow, birds! Look, birds!" they both yelled while pointing to the sky. "Look," David said, "I saw a huge bird!" Mitchell replied, "I saw a huger one!" Mrs. White asked the boys if they would like to record their findings. When they both nodded their heads yes, she handed them each a clipboard, paper and pencil. They got to work. Mitchell leaned over to David and said excitedly, "I saw five birds, I saw five birds!" He noticed that David was sketching a bird and told his friend, "You don't have to draw them all. You don't have to draw that bird, see? You can just put an X. I have five X's." Mitchell held out his clipboard and paper for David to see (though David was very busy with his documentation and did not pay attention). Mrs. White noticed that rather than draw the birds literally, Mitchell had used symbols to represent each bird. He had drawn five "X's" on his paper, one for each bird (one-to-one correspondence). David drew a bird (a representative depiction), and told his friend, "I counted five, too!" (Joyce White, February 25, 2013).

Materials: Binoculars, Clipboard, Paper, Pencil, Birds flying overhead

Shapes in Snow

On a cold (10 degree), drizzly day Louis began putting his snow gear on to go outside. The teachers tried to discourage him because it was so cold, and he began crying. Since numbers were down that day, and they had a student teacher and a parent volunteer, Mrs. Miller was able to take Louis out. When they got outdoors, Louis tested out the sidewalk ("It's not really slick out here."). Mrs. Miller noticed something interesting and pointed out a shape to him on the Climbing/Crawling structure. It had been made by the snow sticking to it. "Louis, look," she said, "What shape is it? It has one, two, three sides." "A triangle," Louis announced. Louis began searching for shapes and found an "upside down L" on the slide. Oliver came outdoors to join them and began climbing up on the structure. He noticed shapes without any prompting. "An X," he told Mrs. Miller… "lines just like we know, just what we were talking about every day!" Then he noticed the shape Mrs. Miller had pointed out to Louis: "Look, a triangle!" "I know," Louis replied, “Mrs. Miller and I saw that.” Oliver kept looking for shapes in the snow and discovered “squares” and “rectangles.” Then he announced, “Mrs. Miller, check it out! The poles are covered in ice!” (Sherry Miller, January 31, 2011).

Materials: Snow (on Climbing/Crawling structure)

Let’s See How Big You Are

Carrie picked up a long stick and held it parallel to her body, vertically, to measure it. She told her teacher: “It’s as tall as me.” She took the stick over to her teacher and said, “If I tried this stick next to you, it would be so small.” Megan noticed what her friend was doing and searched for another stick – she selected a long one. She picked it up and went over to her teacher and said, “Let’s see how big you are.” Megan held the stick up (vertically) next to Mrs. Heinzman and exclaimed, “It’s as tall as you are!” Mrs. Heinzman noted that Megan had selected a stick that WAS the very same height as her (Cindy Heinzman, February 14, 2011).

Materials: A short stick and a long stick

Moving Recycled Christmas Trees

After the holiday break, five four-and-five year old children worked together to drag donated live Christmas trees from one end of the outdoor classroom (at the bottom of the steps) to the sidewalk by the Greenhouse. They worked carefully to negotiate their steps due to a little ice on the stairs. Once they finished, their teacher asked them to talk about their experience. Charlie said, “I remember that big tree was really hard to move. I could not carry it all by myself.” Ned agreed: “It was really, really hard.” Alice added, “They were very, very heavy but the little ones were not heavy. The little ones we could easily carry. We actually got them all up there. We carried them up the steps and we didn’t slip.” Christopher recalled: “We moved all those trees. It was kind of a slick spot. We were putting them on the brick, then on the woodchips. Then we went in to have snack. The small tree, I could carry it all by myself. When we came back outside, I helped Ben move the trees to the Messy Area" (Cindy Heinzman, January 7, 2011).

Materials: Recycled Christmas trees (different sizes)

Now We Are Sliding!

George (3 years, 10 months) gathered small ice chunks from the entire Nature Explore Classroom. He walked all over the space looking for a particular size for his plan–he wanted small chunks that would fit inside a piece of plastic gutter. He propped the three-foot piece of gutter up against a cupboard at an angle to create a ramp. He tried sliding ice chunks down the ramp and experimented with the angle until he got it just right. His teacher observed him talking to the ice chunks: "Now we are sliding!" and asked George, "What did you do with the gutter?" He explained, "We slide them (the ice chunks) down like this" (as he showed the maneuver through the air with his mittened hand). Mrs. V. asked, "How many chunks?" George looked at his mittened hands trying to visualize his fingers as he counted the ice chunks…then he proclaimed "Ten!" (Kris Van Laningham, January 7, 2013).

Materials: Ice chunks, A piece of plastic gutter

We Have Magic Seeds!

Children were engaged in pretend play in the Nature Art Area when Caleb announced, "We have magic seeds!" (small seeds they had removed from a milkweed plant). As they played they continued to take apart milkweed pods to collect seeds. Caleb told his friends, "Show her (Mrs. White) the magic seeds." Landon held the seeds out in his hand and counted, "1, 2, 3, 4, 5, 6, 7" magic seeds. Ellie arrived with seeds and the boys wondered where she got them. Mrs. White suggested they ask her, and she led them to an area in the garden where the milkweed pods were growing. She opened a pod and began blowing seeds out. "Mrs. White, I'm going to the top of the Climbing/Crawling structure with these," Ellie announced with excitement as she grabbed a handful of milkweed pods. Her friend Kailey said, "I want one, I want one!" Ellie took her over to the plant to look for pods. A few minutes later Ellie exclaimed, "Look what I did Mrs. White!" Her seeds went high in the air and stayed up for a long time as she blew them off the Climbing/Crawling structure. Mrs. White asked the children, "What are magic seeds good for?" Caleb replied, "They grow into bean stalks; yeah, that's what my seeds are for." Landon said, "Is that what your seeds will do? I think I will just keep my magic seeds." The children speculated about who had more seeds, but they were difficult to count because they kept blowing away. Caleb kept his seeds in half of a pod, and Landon kept his safe in his hand. The children decided it was "impossible" to figure out who had more seeds just by sight, but Caleb told Landon that he thought he had "more." Caleb tried to decide where he would plant his magic seeds, and said, "It will take me to the clouds. If you have time to climb up and down the beanstalk, you will have a wonderful day!" (Joyce White, November 8, 2012).

Materials: Milkweed Plant, Pods, Seeds

Statues For Sale!

Four-year-old Jonas was playing in the Messy Materials Area when he called Ms. Logan over. She noticed he was standing behind the wooden chest and asked what he had. "Statues for sale!" Jonas replied. "Statues? Where?" Ms. Logan asked. He pointed to a collection of smooth driftwood pieces that were inside the chest. Ms. Logan began pulling them out one by one, asking, "Oh, what's this one?" She pulled eight pieces out and asked Jonas to name each one. Jonas told her the statue she finally bought was from a "Triceratops." Jonas left the area, and so did the teacher, briefly. Then she noticed that three-year old Micah had taken over where Jonas left off. He was "pointing to his merchandise" saying, "Statues for sale, statues for sale!" He offered one to Ms. Logan and asked for one dollar. He said it was the “tummy of a rhinoceros” that was made “a thousand million weeks ago.” He told Ms. Logan that he had sold one to his friend, Gretchen: “It’s a skull from the 1948’s,” and added “That one was two dollars.” (Katie Logan, September 27, 2012).

Materials: Driftwood

Size Comparison – H20 Liquid vs. Solid

Logan (five) found a large piece of ice on the ground. A heart mold was frozen on the top of the ice. He picked it up and carried it around. He needed two hands. Soon he walked over to show his teacher and classmates that it had broken in two. He held one half in each hand, and the heart mold had fallen off. Mrs. White asked Logan a series of questions: “Is it heavy since it is so large? Where was the heart? Are the pieces the same?” Logan replied, “They aren’t the same size, but the heart shape is still on it. It’s not heavy.” He carried the ice into the school (and let it melt). In the afternoon, Logan poured the melted ice into a measuring cup. “It says one (cup),” he read to the class. Then he showed the class with his hands how big the melted water had looked when it was frozen. The children thought there would be more water. Logan placed the measuring cup with the water into the freezer to refreeze. (This was a Thursday, and Logan attended on Tuesdays and Thursdays). When he returned the following Tuesday he removed his experiment from the freezer. He looked at the ice in the measuring cup and told his friends, “It was one (cup), and now it’s more. It gets bigger when it’s frozen.” Mrs. White asked Logan, “Can you find the number two?” “Yeah, there it is,” he replied as he pointed to the line and number on the measuring cup. They discussed that the ice in the cup measured between one and two, and that it was 1½ cups of frozen water. Logan set the ice out to melt one more time (Joyce White, February 19 & 24, 2010).

Materials: Heart mold, Ice, Measuring cup, Freezer

We examined all of the Nature Notes included in this study, plus materials teachers listed on their focus group interview protocol, and constructed lists of materials that supported math learning, categorized by type: 1) Natural, open-ended, loose materials (Table 25); 2) Living organisms (Table 26); and 3) Added tools and materials (Table 27).

Table # 25:
Natural/Open-Ended Loose Materials Identified in Documentation on Math Skills

Ice (various sizes/shapes of chunks, icicles)
Snow
Water
Mud
Corn kernels, cobs, husks
Dirt
Sand
Pinecones
Tree cookies (whole & pieces)
Acorns (whole & pieces)
Leaves/petals (tree and flower)
Tree branches/sticks/twigs
Bark
Willow and bamboo poles (approximately 6' long)
Tree stumps
Driftwood pieces
Wooden planks/slabs
Large geometric-shaped cut chunks of wood/blocks (cubes, rectangles)
Wood chips
Logs
Rocks/Stones (large & small)
Flower seeds (dead-headed, zinnia and marigold)
Seeds/Seed Pods
Decomposing wood
Worm castings/Compost
Bird feathers
Shells
Recycled Christmas trees

Table # 26: Living Organisms Identified in Documentation on Math Skills
Insects (bees, ladybugs, caterpillars, roly poly bugs, red wiggler worms, grasshoppers, aphids, millipedes, centipedes, butterflies) Birds Squirrels Native grasses/Plants Flowers (perennials/annuals) (sunflowers/heads, zinnias, daisies, joe-pye weed, lambs ear, bachelor buttons, coleus, milkweed) Vegetable plants/Herbs (basil, lettuce, Swiss chard, green & purple potted pole beans, pumpkins, squash, corn, tomatoes (multiple varieties), lavender, rosemary, cranberries) Large and mini pumpkins and gourds Dragon Tongue beans/Beans of various varieties/Bean pods Trees (varieties of types/sizes)

Table # 27: Added Materials and Tools Identified in Documentation of Math Skill Development
Collection bowls (wooden and metal of various sizes) Clear plastic containers (various sizes) Buckets (of various sizes) Large tubs Measuring cups/Cups Spoons/Forks/Knives (plastic) Frozen food trays Tea kettle Funnels Scoops Tins (muffin, bread and cake) Clay pots Magnifying glasses Pots and pans Jars Baskets Manipulatives (i.e., tree blocks, mini bricks)

Books
Seed packets
Paint sample cards (to match colors to nature items)
Bird feeder
Planters/Raised beds
Compost bin
Watering cans
Spray bottles/Misters
Scissors
Aquarium
Photos/Visual images of plants
Rulers
Binoculars
Clipboards
Easel
Paints
Chalk
Writing utensils (pencils, markers)
Paper
Plastic open/closed sign with clock with moveable hands
Stopwatch
Large (adult-size) shovels
Child size shovels
Child size rakes (long handled)
Hand trowels/Rakes
Child size hoe
Grinder (to grind corn)
Large push broom
Needle and thread
String
Fabric and clips
Plastic Roly Poly Playground
Scales (floor and postal)
Gutter/Downspout pieces
Scarves
Umbrellas
Slap drum
Nature Art table
Artist's bench
Raised brick edging (to walk on)
Akimbira and mallets
Bricks

<u>The Teacher's Role in Supporting Math Learning in the Nature Explore Classroom</u>

The fourth theme examines the teacher's role in supporting young children's math learning. While the teacher's role is critical in an early childhood setting, we did not initially identify it as a theme. Most of the observations teachers recorded focused on informal, child–initiated experiences (vs. formal, curriculum-based, teacher–directed activities). However, our data clearly illustrate that teachers were instrumental in helping young children think about math principles. This was apparent in the questions they asked children, the materials they provided, the information they shared, the activities they engaged in with children, the invitations they extended to children to document their activities, and the physical assistance they provided (when needed) to help children have rich learning experiences. The following stories illustrate the teacher's role. Tables at the end of this section provide a more comprehensive description of the teacher role.

Matching a Coleus Cutting to the Mother Plant

Five-year old Angela had been helping Ms. Murdoch dead–head plants in the garden when Ms. Murdoch noticed a broken stem of purple basil in the artist garden. She asked Angela to cut the stem off and put it in the rooting tray in the Greenhouse so it could be rooted and grown in a pot in the fall. They went to the Greenhouse together and once Angela had done that, Ms. Murdoch took a rooted coleus cutting from the tray and asked Angela if she could find the "mother plant" in the Artist Garden. Angela took the root with her and went to the garden and matched the cutting to the plant. She decided she wanted to plant the cutting, and dug a hole for it in the Dirt-Digging Area. Once she planted it, Ms. Murdoch suggested Angela measure the coleus and said they would check it again in a week to see how much it grew. Using a clear plastic red ruler, Angela and Ms. Murdoch measured the plant and recorded that it was 7” tall (Holly Murdoch, September 23, 2008).

Teacher Role: Invited child’s participation, gave her a task, joined her, asked if she could match a cutting to a plant in the garden, suggested she measure the plant and measured with her

Digging and Measuring Holes

Five children (three to five years old) were digging holes in the Sand Area. It was a sunny January day, but the sand was sticky and wet from an earlier snow. The children worked with shovels and a hoe. Mrs. White asked Alan, “How heavy is that sand?” He said it was “Sixty pounds” and that he was making his hole “deeper” than his friend Jay’s hole. “This is fun,” he shouted with glee. “I can put my feet in the hole!” Then Erica jumped into the hole she had dug, smiling with great satisfaction and waiting for everyone to acknowledge her accomplishment. Mrs. White suggested children get a ruler to see how deep their holes were. “Yeah, we need to measure!” Erica replied. Erica and Mrs. White went to find the rulers in the Greenhouse. Alan continued digging his hole, using a long-handled metal shovel to dig. Once his hole was about two feet in diameter the width and depth made it more difficult to dig. He wanted to dig deeper, but not wider, so he developed a new way to move dirt with his shovel. He placed his shovel in the hole, bent over it at the waist and pushed on it with the weight of his body as he made it slowly turn, as if he had invented a drill. Curt walked over and picked up a ruler. “When can I measure?” he asked. Looking at his friends’ holes he said, “I think Alan’s is 20 pounds. Maybe Jay’s is deeper,” as he visually compared their holes. At Mrs. White’s request, Curt counted the numbers on the ruler as he measured his hole: “1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12…it’s 12 inches” he told her. Andrea yelled over to her teacher, “Mrs. White, can you measure mine?” Curt ran over and measured Andrea's hole for her. "It's past 12" he told her. "Yay, I'm going to keep digging forever," Andrea sang with enthusiasm as she put her whole body into digging. She let her friend Mandy measure her hole using her body: "Look, her whole leg fits!" Andrea loudly announced. "It comes up to my knees," Mandy said. Andrea began laughing with joy and Mrs. White noticed that Mandy had cradled herself into Andrea's hole. "Look, she fits!" Andrea yelled. Andrea picked up the shovel again and continued to dig. She said with pride, "I'm going to keep digging forever, I hope no one gets mad at me!" Meanwhile, Jay continued to dig, and when he and Alan measured his hole they measured 15 inches. They measured 12 inches, then added on three more inches. Mrs. White showed them how to move the ruler and measure additional inches when the hole was deeper than the length of the ruler. Alan stood on his shovel and continued to dig deeper. Alan and Mrs. White measured again, and this time the hole was 17 inches deep (again they measured 12 inches, moved the ruler and added 5 more inches). Alan stood in his hole and said it was up to his knee (Joyce White, January 6, 2013).

Teacher Role: Asked child about weight, suggested children measure their holes, provided rulers, helped children learn how to measure holes that were deeper than the ruler, measured holes with them

Creating a Bird Feeder

Four-year-old Christopher spent his entire work-time outdoors creating a bird feeder because he told his teacher that "he wanted to be nice to birds." As he was nearing the end of construction on his bird feeder he showed his teacher, pointing to five different places, "Seeds go here, here, here, here, here." Ms. Murdoch said, "You have five places for food." Christopher asked his teacher, "How heavy are birds?" She replied, "Wow, that's a good question!" He explained the reason for his question: "I'm going to make one of those things they sit on. This is gonna be a water place. This is gonna to be a water cup" (he was holding a plastic cup that he had selected from a tub of recyclables). To answer Christopher's question, Ms. Murdoch said, "Maybe they're about as heavy as my hand?" as she rested her hand on Christopher's hand so he could feel the weight. She asked Christopher, "Why do you want to know how heavy birds are?" His reply revealed his thought process: "They might break the stick off." Christopher was quiet and looked like he was thinking so the teacher asked, "What are you thinking?" "I was thinking about where the water should go," Christopher replied. Then he asked, "Do you think a bird can get their head in there?" Ms. Murdoch suggested, "Why don't you try it? Pretend you are the bird." Christopher leaned forward toward the feeder, pretending to be the bird. He finished his feeder and Ms. Murdoch asked him where he wanted to put it. Friends suggested "in the shade" and "in a tree," but Christopher chose to put it on top of the 5' high brick wall, so it would be stable. Since he couldn't reach the wall, Ms. Murdoch lifted him up so he could place his feeder on top. She also helped him put seeds along the top of the wall so birds would be attracted to it and to make sure the child could be successful (Holly Murdoch, September 17, 2008).

Teacher role: Affirmed child's question, asked child thoughtful questions, provided math vocabulary (number), helped child consider the concept of weight, physically assisted child in placing bird feeder up high

Weighing School Supplies

Daniel and Briana (both four) were in the Greenhouse and wanted to weigh school supplies. Daniel placed a pair of child's scissors on the scale. The dial moved to the two-ounce line. Daniel looked closely at the scale and wrinkled his nose. Ms. Murdoch explained, "We can see the one and the three, but not the two. So, it weighs two ounces. When we weigh something big like you we say pounds." Briana added a second pair of scissors to the scale and Daniel read the number “Four.” Daniel picked up a ruler and said, “I want to measure this,” and held the ruler up to the scale. Then both children went back to using the scale. Briana placed several rulers and a few pairs of scissors on the scale. She and Daniel looked closely at the numbers on the dial. The combination of items weighed 12 ounces. Mrs. Murdoch explained: “When something weighs 16 ounces we can say it weighs a pound” (Holly Murdoch, December 13, 2010).

Teacher role: Helped children read the scale, provided math vocabulary (ounces, pounds, weigh), discussed when to use ounces versus pounds

Which One is the Next Biggest? Arranging Leaves by Size

Daniel (five) had collected a large pile of yellow/golden leaves from around the paw paw tree in the outdoor classroom. Ms. Murdoch asked, “Daniel, what color would you call those leaves?” “Yellow,” he replied. “How did you collect those leaves,” Ms. Murdoch asked? Daniel pointed to the paw paw tree and said, “From those there.” Ms. Murdoch wondered, “Do you have a plan for them?” and Daniel said, “They’re just for looking at.” He held up a big leaf and marveled, “There’s some big ones…like this one.” Almost talking to himself he asked, “Is this one the biggest?” Ms. Murdoch asked Daniel, “Do you think you could put them in order (by size)?” “Um hum,” Daniel replied. He asked himself, “Which one is the next biggest? I think that one…um hum…then this one. That one is not the biggest.” Daniel held each leaf up, beside the others to compare them, and lined them up by size. Ms. Murdoch counted 24 leaves in Daniel’s line so far, as he continued to add to it. He rationally counted 27 in the line (i.e., assigning a number to each item as he counted)…then 29 and said to Ms. Murdoch, “I can’t count any farther.” Ms. Murdoch helped with the number “30,” then Daniel was able to continue to count the rest, to 32 (Holly Murdoch, October, 21, 2010).

Teacher Role: Invited child to put leaves in order of size, counted leaves as child was working, helped child identify the number 30 when he was stuck

"I'm Going to Check the Humidity."

Blake (almost four) and his parents often walked to the Nature Explore Classroom in the summer. On the first day of Blake's second year at preschool he went to his teacher, took her hand and led her toward the entrance of the outdoor classroom. He said, "I'm going to check the humidity." They walked to the gate, near where the thermometer was, and he climbed up on a wooden ledge and held onto the wooden shelf. He leaned in just enough so that he could see the thermometer that was about 5½ feet off the ground. Blake looked at the thermometer and said, "It's 40 humidity." The teacher noticed that the needle was pointing to the number 40 in the 'humidity' part of the thermometer and responded, "Yes, it is 40 percent humidity." She commented that the humidity level is often higher in the summer because there is more moisture in the air. Blake looked at the thermometer again, and said, "It's 80 F." "Yes," his teacher replied, "it is 80 degrees Fahrenheit – that's pretty warm." Blake looked at the thermometer again and asked, "When is it going to be 10 humidity?" His teacher said she did not know for sure but that the humidity level often dropped in the winter. He looked again and said, "When is it going to be 20 C?" His teacher explained that "C" stands for Celsius, and that she is not as familiar with Celsius because she uses Fahrenheit, but added that she could tell by looking at the numbers that when it was 20 Celsius it would be cooler outside. Blake asked, "When is it going to be 0 F?" Mrs. Reitz reminded him of some of the coldest days last winter, and told him that those days would have been 0 degrees Fahrenheit. He asked, "When is it going to be 0 C?" She explained that when it is 0 degrees Celsius water freezes into ice. Blake's questions continued and at one point Mrs. Reitz referred to the instrument as a "thermometer." Blake said, "I have a thermometer," as he opened his mouth wide and pointed under his tongue. Then they discussed the two types of thermometers. After a lengthy discussion, Blake jumped down and began to walk toward the Climbing/Crawling structure. He looked up to the top of the church tower where there was a line of pigeons sitting. He said, "Look at all those birds up there. I think the big one is the mommy." He looked at them for a moment, then walked away (Kristi Reitz, August 19, 2010).

Teacher role: Answered child's questions by providing information about number and Celsius and Fahrenheit, provided math vocabulary, encouraged child to consider time (previous winter and future), explained what the numbers meant

"How Long Will it Take Me?" Racing Around the Track!

Caden (five) created a path around the Climbing/Crawling structure by dragging his foot through the woodchips. Shania (three) and Bret (four) joined Caden and the three children ran round and round the track. Caden asked Mrs. White, "How long will it take me?" (to run around the track), and he estimated 10 seconds. Mrs. White counted aloud as Caden raced around the track in seven seconds. Caden timed Mrs. White while she raced around the track. He announced with a huge smile, "It took you 20 seconds to run around the track." Four-year old Lenny joined the others and asked if he could help count. Caden and Lenny took turns timing each other by counting out loud. They came up with several different times (20 seconds, 44 seconds, 8 seconds). Michael (four) stood in the background singing the children's times as the boys announced them (Joyce White, February 14, 2013).

Teacher role: Timed children by counting out loud, participated in activity so child could time her race

“I was Trying to Reach This Branch.”

Mrs. Reitz noticed Michael (four), standing on a large stump that was under a tall, mature tree (near the trunk) in the NEC. The stump was approximately 18” high and 24” in diameter. Michael stood on the stump and reached up with one arm toward the tree, as if trying to reach something. He climbed down and walked to a large (natural) rectangular-shaped tree block, about 12” by 6” by 6”. He picked it up using both arms, carried it over and put it down on top of the stump he had been standing on. Then he got another rectangular tree piece and added it to the stack. Mrs. Reitz asked Michael what he was doing and he replied, “I was trying to reach this branch,” pointing to indicate a place on the tree where the trunk forked into two smaller extensions about 7 feet off the ground. Michael got a third rectangular tree piece and put it on top of the stack with Mrs. Reitz’s help. He climbed onto the stump, then put his hands on the slightly smaller stack of blocks, getting ready to climb higher. Mrs. Reitz was right there to stop him if he was in danger. Michael noticed that the stacked wood pieces were wobbly. He removed the fourth/top block, with help from Mrs. Reitz. He again attempted to climb and realized the stack was still wobbly. He told his teacher, “Actually I only need two,” and took the third block off. He got on the stump, put one foot on the top of the two blocks, reached and realized he was nowhere near the branch that he wanted to touch. He got down and put the third block back on the stack. He attempted to climb (again, Mrs. Reitz was right there to make sure he was safe), and even though it was still wobbly, he got down and added the fourth block back on top with help. Mrs. Reitz asked, “Do you want me to hold it while you climb?” Michael said, “yes” so she braced the blocks and held onto him. He climbed just to the edge of the second block when he wanted to get down. Still determined to reach the branch, he changed his plan. “I’ll just reach it with a stick,” he said and went and found a long stick. He climbed up again to the second block with Mrs. Reitz's support, reached up toward the tree with the stick, and touched the left limb of the tree branch. He came down and was finished. He and Mrs. Reitz took the large blocks down, as they discussed whether it would have been safe to do the activity without an adult's help (Kristi Reitz, February 5, 2009).

Teacher Role: Provided physical assistance so he could pursue his plan, allowed him to take risks and problem solve while monitoring for safety

"The Wet Beans are Bigger!" Observing and Sorting Beans

During small group time in the Greenhouse, Mrs. Tichota gave each child a variety of dry bean seeds and some corn. The bean seeds were different sizes, types and colors. Immediately many of the children began to sort the beans by type as they observed their characteristics: "They're different sizes." "They're hard." "They're different colors." Children noticed the larger and smaller spots on each bean. Next, Mrs. Tichota gave the children seeds that had been soaked in water overnight. One child said, "They (the wet seeds) are bigger. They're full of water." The children had a discussion about why they wear coats and the purpose of seed coats and began peeling the seed coats off the soaked beans. The children also noticed that the beans broke into two pieces. Mrs. Tichota told them this is where food is stored for the new little plant (they broke along the thin line on the bean). One child asked if they could eat the beans. Mrs. Tichota said,"We do eat beans, but we cook them first" (Kathy Tichota, February 29, 2008).

Teacher Role: Brought interesting materials that children could compare (dry beans vs. beans soaked in water)

Making Food for a Witch's Halloween Party

Baker was playing along side Chloe, Patty and Laura in the Sand Area. The children said they were making food for a witch's Halloween party. Baker was using a hand shovel to fill a large bowl with sand. Patty said she was making worm cake for the witches to eat. Baker said, "I'm making evil cookies." Baker announced to Patty, "The cookies are done. I put the cookies on the evil table" (a tree stump in the Sand Area). Baker added a handful of pine needles to Patty's worm cake and said, "Here's some more worms." Baker started shoveling sand into a muffin tin. He said, "I'm making some witches' cupcakes." Mrs. Tichota asked Baker, "What do you put in your witches cupcakes? I'd like to write down your recipe." Baker replied, "one cake batter, sugar, some evil frosting, 100 pumpkins and one evil leaf." Baker placed a large yellow leaf from the paw paw tree on top of his cupcakes. Mrs. Tichota asked, "How long do you bake your cupcakes?" Baker answered, "Bake for 100 pumpkin minutes" (Kathy Tichota, October 11, 2011).

Teacher Role: Asked child open-ended questions that encouraged him to classify ingredients in a recipe and to consider a time concept (how long to bake)

Measuring Dragon Tongue Beans

Daniel and two of his friends harvested Dragon Tongue Beans from the raised planter. They traced their beans, measured the size of the beans with rulers, and compared the length of their beans. Daniel said to Mrs. White, "It's five (inches), can you write five?" Instead of writing the whole number on Daniel's paper, Mrs. White asked Daniel to make a short, vertical line and she added the "humpity bump" on the numeral. Then Daniel made the rest of the number. Once he had an example, he made an almost identical number just to the left of the one Mrs. White had assisted him with (Joyce White, May 10, 2010).

Teacher Role: Assisted child writing the number five (did not do it for him as he requested)

These examples provide insight into the ways teachers supported children's math learning, especially in activities that children initiated based on their interests. Table 28 identifies the kinds of questions teachers asked children to encourage mathematical thinking. Table 29 illustrates a variety of ways teachers supported math learning by providing children with specific materials, information, and/or vocabulary during their play. Table 30 describes several ways teachers helped and assisted children in pursuing interests that led to math-related thinking and discussions. Table 31 identifies some of the requests and invitations teachers extended to children that fostered math thinking.

Table # 28: Questions Teachers Asked That Supported Children's Mathematical Thinking

Questions teachers asked:	**Mathematical Concept(s)**
How heavy is it?	Estimation/weight
Is it a triangle?	Geometric shape
How long do you think it is?	Measurement/length
Which is longer, the stick or the water feature?	Measurement/length & comparison
How big is it?	Size
Should we get a piece of paper to trace it?	3-D to 2-D
How many seeds do you think are in one of these?	Number/estimation
How many seeds have you planted?	Number
That is a tall plant. How many feet tall do you think it is?	Estimation/height
What do you put in your witches' cupcakes? How long do you bake your cupcakes?	Classification of ingredients & time
Can you take a scientific guess – how many seeds (hyacinth beans) have you collected?	Estimation/number
What did you do first?	Sequencing
What color are these leaves?	Classification/color
Do you think you could put them (leaves) in order?	Seriation
How big do you think that (hole) is?	Size
I wonder how many pumpkins there are?	Number
How many pizzas have you sold? (Later) How many have you sold now?	Number/addition
How heavy is the kettle of water?	Estimation/weight
I wonder how many you have now? How many more do you need? (Counting scarves)	Number
Wow, I wonder how many there are?	Number
Which one do you think is the largest? (comparing the size of	Estimation/size

corn cobs)	comparison
What are the numbers – how much do you weigh?	Number/weight
Why did you take that piece away from the bridge? (child removed triangular-shaped piece)	Subtraction/ geometric shape
It has 1, 2, 3 sides – what shape is it?	Geometric shape/number
How many bricks were in the circle?	Number/geometric shape
How many chunks (of snow did child place in a piece of gutter)?	Number
How do you know the word "equal"?	Quantity
Can you make something large enough that all your bodies will fit inside?	Volume/size
I wonder why there would be an old bug body in there, and how it got there?	Time concept

Table # 29: Ways Teachers "Provided" Materials, Information and Vocabulary to Support Children's Mathematical Thinking

Teachers Provided:	**Mathematical Concept(s)**
• information regarding the weight of a bird when a child who made a bird feeder was trying to determine the size of perch he needed to add, and asked the teacher, "How heavy are birds?" (teacher replied: "Maybe as heavy as my hand" and placed her hand on the child's hand)	Weight/size
• materials for a child to sketch an observation in the NEC	3-D to 2-D/scale relationships
• information about shapes (& form) to prepare children to sketch insects they had observed	Geometric shapes
• a number chart to help children document the number of pumpkins they had counted	Number/counting
• small scale miniature bricks so a child could build a small scale brick wall, imitating the work of a sculptor in the NEC who created a large scale brick wall in the Messy Materials Area	Scale relationships
• dotted numbers on a piece of paper, in the shape of the numeral two, so child could connect the dots to create the number (as a follow-up to a counting activity)	Number/counting
• information regarding how to prop up a number of 6' long poles at the correct angle to make a 100% scale pyramid-shaped structure children could fit inside (so poles reached from the ground to the tree branches)	Angle, scale relationships, length
• the vocabulary word, "line" when a 3 year old boy asked his teacher to observe what he had made (i.e., he made 3 lines – one in the Messy Materials Area by dragging his body through wood chips, one in the wood chips near the Climbing/Crawling	Lines/math vocabulary/size comparison/3-D to 2-D

structure using a stick, and one smaller scale line on a piece of paper using a pencil)	
• multiple varieties of plants for children to observe, count and classify by attribute	Number/counting & classification
• information about time concepts (i.e., when engaged in an activity where children were dead-heading zinnas to store for the following year)	Time
• numerical vocabulary for a child by teaching her a counting song about five green speckled frogs (which she translated to her play by acting out the song with her whole body)	Number/math vocabulary
• mathematical directional words for a three year old boy as they walked the garden pathways (i.e., way up, way down, on the left, on the right)	Math vocabulary/ directionality
• information and math vocabulary to children who were planting seeds (i.e., to fill their cups with soil, then plant 3 seeds in the shape of a triangle)	Math vocabulary/ volume, sequencing, geometric shape, number
• a bird-watching observation space to engage children in counting, comparing, classifying and sketching birds	Number/3-D to 2-D/ Classification
• photo and word label markers that children could stick in the ground (once they matched the plant on the marker to the same plant in the NEC)	Matching
• binoculars, clipboards, paper and pencils for children to sketch their observations (birds)	Distance/ 3-D to 2-D
• additional dirt in the planting box as children mixed soil and compost	Volume/quantity/ addition
• rulers, pencils and paper so children could measure and document beans before eating them	Measurement/ 3-D to 2-D/shape/number
• materials so children could count and classify hyacinth beans in the Greenhouse	Classification/ counting/number
• a ruler to a child so he could measure a hole he dug in the sand	Measurement
• information about the sequence of the life-cycle of plants	Sequencing/time
• the number "five" when a child pointed to five places on his bird feeder that would hold food (saying, "here," "here," "here," "here," "here" as he touched each of the places) (teacher said: "you have five places for food")	Number/one-to-one correspondence

Table # 30: Ways Teachers Helped or Assisted Children That Supported Mathematical Thinking

Teachers helped/assisted children by:	Mathematical Concept(s)
• showing a child how to make a circle in the center of a large rectangular space and count off five feet on either side (for a game)	Geometric shapes/center/ counting
• showing a child how to make tally marks to keep score	Number/counting
• showing a child how to begin a game of hockey by counting off	Number/counting

with sticks, “1, 2, 3, 4”	
• showing a child how to read numbers on a scale and understand the differences between ounces and pounds	Number/weight
• encouraging a child to count with her (“Let’s count”), then using fingers as she counted with the child to 15	Number/counting/ one-to-one correspondence
• counting seeds with children, to 63	Number/counting
• reading a postal scale for a child who was weighing small pumpkins (“It weighs one and a half pounds.”)	Number/weight
• sketching around a child’s hands so he could add the number of acorns he collected to each hand drawn on the paper, then drawing a bucket so the child could make a line to indicate how full the bucket was (teacher held the acorns as the child sketched so he could see the size and quantity)	3-D to 2-D, number/ quantity/size
• showing children where to find rulers in the Greenhouse, then how to measure the depth of holes they dug in the sand (when the holes were longer than the rulers)	Measurement/depth
• challenging a group of children to break their record and find more than 25 roly polys (found earlier in the day); then counting 105 roly polys with children (and by opening a container of soil that had many more roly polys to count)	Number/counting
• counting out loud as she pointed to each key on the akimbira, to confirm there were 11 notes (as child observed)	Number/counting
• asking child to document her collection of nature items (child categorized while sketching and numbered eight different types of nature items – teacher listed the items beside the child’s numbers)	3-D to 2-D, classification by attribute, number
• suggesting a child use a stick to measure deep holes in snow and illustrating how to do it	Measurement/depth
• encouraging a child to draw around the end of a stick that had a triangular-shaped end (once the child drew the shape at the end of the stick the teacher drew an isosceles triangle on top of the sketch to illustrate and match the shape)	3-D to 2-D, geometric shape
• holding a child’s hand to steady it as he traced around a leaf to document its size (at his request: “Can you help me?”)	Size
• counting how many seconds it took for a child to run an obstacle course (multiple times) (at his request)	Number/counting/ time/ repeated pattern/speed
• sorting natural materials by attribute with children (corn cobs, kernels, sand)	Classification/sorting
• counting and classifying petals, seeds and plants with children	Number/counting/ classification
• collecting live creatures with children so they could bring those into the classroom, to place in an aquarium to observe, count, and classify	Classification/ number/counting
• physically assisting a child who needed help stacking large rectangular-shaped blocks of wood to get to the right height to	Estimation/height/ addition/subtraction

stand on the stack and reach a tree branch (child experimented adding and subtracting pieces to find the right height)	
• showing child how to use a ruler to measure a young plant (and telling the child that they would measure the plant again in a week to see how much it grew)	Measurement/height
• showing child how to find specific information by looking it up in a reference book, in order to classify types of insects child had observed in the NEC	Classification

Table # 31: Requests and Invitations Teachers Made to Support Children's Mathematical Thinking

Teachers requested or invited children to:	Mathematical Concept(s):
• participate in a "survey" to determine how many had appropriate clothing and were prepared to go outdoors on a snowy winter day (teacher created two columns on a clipboard and surveyed children, asking children to add the columns and compare the numbers)	Number/counting
• sample three varieties of lettuce growing in the garden, then discuss each variety based on its properties (shape, color, texture, taste)	Number/shape/ classification
• sketch the varieties of lettuce growing in the garden	3-D to 2-D/ classification
• go outdoors to see if they could find a similar bug that matched an orange ladybug they found in the aquarium indoors	Matching/ classification
• create maps based on their experiences in the NEC	3-D to 2-D/spatial relationships/lines/ geometric shapes
• take long planks of wood from the indoor block area to the NEC to create incline planes that they could roll small cars down (children compared which cars went the fastest and farthest, contrasted the amount of force needed to push the cars, estimated the angle of the ramp in relation to the speed the cars traveled, and estimated the length of the plank in order to create secure, angled ramps)	Estimation/angle/ speed/length/distance
• design and implement a "system" children were interested in creating; i.e., a process to make snow cones, then to recall and sequence their actions (Children created a snow cone machine using acorn lids as cups and snow as shaved ice, and colored pencils to create flavors)	Sequencing/volume/ classification by color/math vocabulary
• match seed packets to potted plants growing in the Greenhouse	Matching
• match the rooted coleus plant in the Greenhouse to the "mother plant" in the garden	Matching
• recall a specific event in the NEC that encouraged a child to tell	Sequencing

a story verbally sequencing events (describing his process of creating a forest using sticks and leaves in the NEC)	
• use chalk to write numbers on stumps they had used to create an obstacle course	Number

Discussion

No one disputes the role children's earliest years play in their brain and body development. Researchers have well established that for young children the early years are formative. Even in those first few years, young children, who are concrete thinkers, are capable of constructing math knowledge and developing skills that provide a solid foundation for later math learning. Our findings illustrate the richness of one early education program's intentionally designed outdoor classroom in supporting math learning in children's formative years.

Researchers have concluded that the learning that occurs in preschool settings can be parlayed into success in school in later years – in math and in other subjects. According to Edward Melhuish, a Professor of Human Development at the University of London, children who attended preschool performed better in math at age 10 than classmates who did not have a preschool education. Melhuish's team analyzed data from over 2,500 children who attended preschool for an average of 18 months, and their findings indicated that not only did preschool boost children's cognitive and social development, but when children began school "many areas of their development (were) better than their peers who did not attend preschool, including math scores." In fact, the average 10 year old who attended preschool scored 27% higher on a standard math test than comparable peers who did not attend preschool (cited in Ostrow, August 2008). Early learning in preschool was directly correlated with academic success in the elementary years. While our research was exploratory and we have not conducted longitudinal research on children who left Dimensions Early Education Programs, teachers have received anecdotal evidence from parents who have said their children performed well in math in elementary school (and attributed that success to the quality of their early education). This would be an interesting line of inquiry to pursue in future research.

Since ours was the first study that specifically explored how children were developing math skills in a Nature Explore Classroom, our goal was to understand what that learning looked like for young children, and how early math learning could best be supported. Our research was open-ended, and as we analyzed teachers' documentation, salient themes emerged. We described those themes (math skills, math vocabulary, materials, and the teacher's role) at length in the Findings section. As we contemplated the best way to "make sense" of our data for readers we decided to use a "lessons learned" approach. In their hallmark qualitative text *Naturalistic Inquiry*, Lincoln and Guba (1985) suggested that in order to craft a "substantive case report" researchers needed to include several elements: an explication of the problem or issue being studied, a thorough description of the setting and context of the study, a thorough description of the salient findings that were important (using "thick description"), and "lessons to be learned" from the study (p. 362). This is what we call the "So What?" of our research – in other words…so what did we learn from the observations our teacher/co-researchers recorded? To share our interpretations, we have framed our discussion around eight key lessons that can be gleaned from this study

Lesson # 1: The observations we shared, drawn directly from teachers' Nature Notes, and the data represented in Tables 3–17 provide clear evidence that young (preschool/kindergarten-age) children were developing foundational early math skills through their play experiences in the Nature Explore Classroom.

Our findings support earlier studies that concluded that young children are, by nature, curious about math and that the goal in working with young children is to provide them with hands-on experiences that will help them develop a solid foundation for later, more complex learning. Their process of knowledge construction is experiential–math becomes real to children as they use it by talking, reasoning, playing and doing (Stanberry, 2014). This study provides research-based evidence that children were developing math thinking and skills through their actions and interactions in the outdoor classroom.

Teacher's observations illustrate that even at a young age, children were learning math skills that were more complex than the basic concepts of number, counting, shape, size and volume. In addition to these concepts, children learned about pattern and symmetry, dimensionality, and positive and negative space. They learned to classify materials based on a variety of attributes and had opportunities to use computational skills in their play. Because of the richness and uniqueness of the outdoor environment, children learned about scale relationships and whole-part relationships. Because of the changing and growing nature of the outdoor environment, children learned about time concepts. Teachers' Nature Notes illustrate that children had many opportunities to learn about math concepts such as estimation, comparison, measurement, sequencing and seriation. They developed vocabulary for the math concepts they were learning and learned how to use tools and materials that extended their math thinking, such as rulers, scales, and thermometers. They had many opportunities to hone their observation skills as they collected data that helped them make sense of the world.

Our findings indicate that a number of influences contributed to children's math learning (which we will discuss in greater depth later in this section). These influences include:

• children's intrinsic motivation to learn in ways that were meaningful to them, through play and pursuing their interests and plans;

• children's curiosity about math concepts and how the world works;

• the time children had to explore the rich, natural environment in an outdoor space that was intentionally designed to support skill development (in math and other domains);

• the meaningful social interactions children had with peers and teachers;

• the physical experiences (sensory, tactile, whole body) that children had in a large, outdoor space and with a variety of interesting materials (natural, made, living and growing);

• skilled and caring teachers who carefully observed and listened to children, recognized teachable moments and provided age-appropriate information at those times, asked questions that encouraged math thinking, invited children to participate in experiences that intentionally extended their math thinking, and validated children by their enthusiasm, by participating in children's play, and by documenting their learning.

While the focus of this research was on math skills, our data suggest that children do not develop those skills in isolation. While children developed math skills, they were simultaneously developing other skills, including visual-spatial skills, social skills, kinesthetic skills, intrapersonal skills, science skills, construction and engineering skills, and language and literacy skills. In reality, children's math learning was part of much more holistic (i.e., whole–child) learning experiences. We could easily have taken the same data (i.e., the 148 pieces of documentation we analyzed for this study) and examined them using all of the other skills as a lens. The value of this kind of integrated learning is that it is not narrow and isolated by subject. This makes math skills more relevant to everyday life. Interestingly, adults tend to view math as a cognitive exercise but our research and other's studies confirm that it is much more than that. Waite-Stupiansky and Stupiansky (1992) suggested that while "math stretches young children cognitively…it also enhances every other area of the whole child. It challenges the child's creative thinking skills. It can be a physical activity that develops fine-and gross-motor skills…(and) math experiences help hone essential social skills…success as a math thinker helps children reach emotional milestones, like developing self–confidence" (p. 9). This was evident in the stories we shared.

Lesson # 2: Our research supports the value of PLAY as an age-appropriate and meaningful vehicle for children to learn math concepts.

Learning should be fun, even joyful…and for young children, learning occurs through something they know and do naturally–play. There is much debate about how to "teach math," even at the preschool level. The primary consideration should be to provide learning experiences that make sense to children, are engaging, help them develop a love for learning, and develop confidence as learners. The stories we shared in the Findings section illustrate this kind of joyful play.

Teachers' observations documented children at play, primarily self-initiated play, and the rich learning that occurred through that play. Children explored and experimented; used their imaginations; used materials in interesting, even unconventional ways; collected items that were interesting to them; and constructed structures that they could play on and inside. They were motivated to move heavy materials to pursue their plans. They made elaborate recipes, sold interesting items, and assumed a variety of roles in their pretend play. They were completely immersed in their play and were not deterred by inclement weather conditions. Many children stayed focused on their play for significant amounts of time, and returned to the same play experiences repeatedly because they were meaningful to them.

Our study supports several authors' contentions that play IS learning, and is the most appropriate way to "teach" math to young children. For example, Waite-Stupiansky and Stupiansky (1992) suggested that:

> Math learning happens naturally as children play…Young children discover, test, and apply math concepts naturally everyday in just about everything they do…children…are "doing math" as they discuss whose cup is biggest or which bucket holds the most sand. They are developing problem-solving skills by working through playtime dilemmas, such as deciding which size block will make the best roof for a building…Young children need time to explore and discover math concepts on their own. In a supportive, non-judgmental environment, they become confident learners who see math as part of everyday activities. Most important…these skills are best developed in natural, meaningful settings, using concrete materials that children can manipulate and explore on their own. These are not skills to "teach." They are skills for children to discover and refine through everyday play (pp. 6–10).

Similarly, Clements (2001) suggested that, "Children's play and interests are the sources of their first mathematical experiences" (p. 272), and Richardson (undated) indicated that children's math understanding is best "developed through stories, songs, games and imaginative play" (p. 6). Several authors have suggested that learning should be enjoyable. When it comes to helping young children learn math, specifically, Clements (2001) said:

> High quality teaching in mathematics is about challenge and joy, not imposition and pressure. Good early childhood mathematics is broader and deeper than mere practice in counting and adding. It includes debating which child is bigger and drawing maps to treasure buried outside…Quality preschool mathematics is not elementary arithmetic

pushed onto younger children. Instead, it invites children to experience mathematics as they play in, describe, and think about their world (p. 270).

In considering the process of constructing math knowledge in the early years, Elkind (1999) suggested that, “even without explicit instruction, young children are acquiring elementary and adaptive knowledge and skills in math” (p. 3). Whitin, Mills and O’Keefe (1991) proposed that through the vehicle of play, children tend to naturally develop math skills “as they create meaning for the stories they wish to communicate” (p. 19).

Children create meaning collaboratively and many of the experiences teachers documented in their Nature Notes described shared experiences. Shared communication in the context of play is a powerful teacher. Whitin, Mills and O’Keefe (1991) described learning math as “a collaborative adventure” because “children do not learn (math) apart from their social world” (p. 6). They proposed that “authentic math experiences encourage social interaction” (p. 17) and that when children have the opportunity “to use each other as resources, (they) extend and refine each other’s understanding” (p. 107). As children worked with each other in the outdoor classroom, they observed, listened to and learned from their peers’ knowledge and discoveries. As children shared their math observations with friends, they not only had the opportunity to refine their understanding of math concepts, but also to teach their peers.

Lesson # 3: Our findings illustrate the role of purposeful movement in constructing early math knowledge, as children manipulated materials and physically experienced the math concepts they were learning.

Children’s math learning was richer because of the physical experiences that accompanied their play. In traditional (indoor) classrooms, teachers use small manipulatives to teach math concepts. Their “hands-on” math lessons consist primarily of fine motor experiences that are relatively limited because of space constraints. Because children at Dimensions had daily opportunities to go outdoors, they used their whole-bodies and physically experienced the math concepts they were learning (e.g., weight, height, length, distance, whole-part relationships, scale relationships, size comparisons). Teachers' Nature Notes illustrated how purposeful children's movements were as they pursued their plans. Sixty-six percent (n=97) of teachers' observations documented gross motor experiences. Many of the observations we analyzed described whole-body experiences that happened specifically because of the possibilities provided by the space and materials available in the outdoor classroom. We documented children's large motor movements as we examined each observation. The gross motor, whole body movement children were engaged in (that related specifically to math learning) included:

- traveling through the Nature Explore Classroom to collect materials and pursue their plans,
- lifting, carrying, and placing a variety of objects (different sizes and weights),
- balancing objects and their bodies,
- walking (experiencing pathways, perimeters, slopes),
- running and racing,
- hopping and jumping (in space and onto and off of objects),
- climbing (experiencing different heights/levels),
- using tools with their bodies (digging with shovels, using hoes, rakes),
- pushing, pulling and dragging objects (various sizes and weights),
- dumping and filling containers of various sizes (large bowls, buckets),
- stepping through depths of snow and negotiating slippery surfaces,
- constructing with large materials and a variety of shapes,
- reaching and stretching to extend their bodies, and

• crawling (over, under, through objects).

Forty-eight percent (n=71) of the Nature Notes we analyzed also identified fine motor skills children were using in their math-related play. Some of those math-related experiences included:

• picking up fragile materials carefully,

• counting and/or sorting small items such as beans and seeds,

• planting seeds in small containers,

• holding insects, worms, and roly polys gently,

• dead-heading plants and cutting flowers with scissors,

• handling small objects (e.g., sorting/counting small kernels of corn, removing seed coats from beans, removing corn kernels from cobs, planting seeds),

• cutting lettuce leaves with small plastic knives, and

• sketching, writing words, numbers, symbols, lines, and shapes on paper (typically related to children's observations and the play experiences they wanted to document).

The physicality of children's activities in the outdoor classroom allowed the concepts they were learning to become more real to them, because they were learning experientially. Literal, hands-on experiences are important for young children because they are concrete thinkers. Unlike older children and adults "who can use abstract methods such as paper-and-pencil arithmetic or mental calculations to solve math problems," the High/Scope Foundation (2003) suggested that "young children first need to see objects and physically work with them before coming to their mathematical conclusions." Those were precisely the kinds of experiences children were engaged in in the outdoor classroom. Waite-Stupiansky and Stupiansky (1992) similarly indicated that, "As children touch, pour, shape, and order materials around them, they discover relationships among objects…(and) each new discovery about the physical world, and the thinking that accompanies these discoveries, lays the foundation for later mathematical learning" (p. 7).

Lesson # 4: Our findings illustrate that children had many opportunities to learn, practice and share math vocabulary through their play in the outdoor classroom. Teacher support and a variety of interesting materials contributed to the use and richness of children's math vocabulary.

The tables in the Findings section illustrate the variety of the math vocabulary children used in their play outdoors. That vocabulary was attached to experiences and discoveries that had meaning for children. They used vocabulary contextually, in their play, to communicate information, share stories, and/or ask questions that were relevant to them, and they were excited to verbalize their knowledge and speculations to teachers and peers. Their vocabulary ranged from simple to complex. While some children's vocabulary reflected concepts they had previously learned, sharing their knowledge in different contexts (attached to new experiences and materials) gave them opportunities to further their understanding of math principles involved in their play. According to Waite-Stupiansky and Stupiansky (1992) this is important, because "Each time children put their math thinking into words to share with others, they further their understanding of math concepts and the purpose of language" (p. 21).

Children used math vocabulary to: describe attributes of materials; identify geometric shapes; compare and contemplate size concepts; estimate and compare distances, heights, lengths, weights; describe changes in volume; count and tally items; and to sequence actions (to name a few uses). Children sometimes used math vocabulary to link to other contexts, such as experiences they recalled in the past, at home or in other settings (often related to time concepts).

Some children used words specific to math processes, such as using “rulers,” “measuring,” doing “math problems,” calculating “inches,” and used math terms like “half” and “equals.” Sometimes children built on each other’s vocabulary. For example, when three boys were sorting acorns they began describing their sizes by assigning attributes to them. The first boy said a small acorn was “a baby,” the second boy said a larger acorn was “a junior,” and a third boy said another acorn was “a two-year-old.” Children were not only communicating their understanding of size relationships, but potentially learning from the vocabulary their peers used.

Brickman and Taylor (1991) suggested that as young children construct math knowledge they experiment with vocabulary. For example, the authors explained that initially children describe amounts and quantities in gross terms (a lot, more than, bigger, smaller, a little bit) before they are able to accurately judge quantities by counting. This general use of vocabulary is a valuable precursor to understanding number and quantity concepts later. The vocabulary tables we shared in the Findings section illustrate this–children used gross terms in many of their verbalizations. Their math vocabulary included words and phrases like: “a whole, whole bunch,” “you need some more,” “add more stuff,” “maybe there is less,” “huge” and “huger,” “really, really, really tiny,” and “I have a lot, he has a little.” Yet, some children were very precise in their use of math vocabulary (e.g., adding and subtracting accurately, accurately communicating observations such as, “It has three plus three on both sides and two antennas.”). The three-to-six year-old children we studied were at very different places developmentally, but what was most important was that they had many opportunities to think about and talk about math concepts with teachers and their peers, while they were engaged in activities that excited them.

It is important to note that in some instances, children were experimenting with math vocabulary, and did not always have the accurate words. This is characteristic of where young children are at developmentally. Brickman and Taylor (1991) suggested that children do not always begin with the "correct" vocabulary. For example, children begin by using visual estimates rather than precise measurements. By estimating and experimenting, they solve problems such as deciding the length (span) they need to build a bridge, the depth of a hole they created, or how many items they collected. They do this before they learn to use rulers, and before they necessarily have the precise vocabulary to communicate their observations. They learn about quantity and volume through simple dump and fill activities, before they have the vocabulary to describe quantities. Through these kinds of active experiences, "Children develop the foundations for more advanced math skills," and later develop the vocabulary to describe them (p. 92). In order for children "…to operate successfully in the world, (they) must (first) learn the concepts of light and heavy, behind, inside and on top, night and day, before and after, and much, much more…Perhaps Friedrich Froebel (1904) the inventor of kindergarten, put it best when he wrote that young children need to 'learn the language of forms before they learn the language of words."

Richardson (undated) described the importance of giving children "sufficient time, space and encouragement to use 'new' words and mathematical ideas, concepts and language during child-initiated activities in their own play" (p. 6). One way teachers encouraged children's emerging understanding of math concepts was through the vocabulary they used as they questioned and/or responded to children (e.g., discussing the difference between ounces and pounds, or Fahrenheit and Celsius, and providing vocabulary words such as "scale" and "thermometer") and by assisting children at teachable moments when they became stuck (e.g., providing a number so children could continue counting).

Listening to children's vocabulary provided insight into their math knowledge. For example, a four-year-old girl retrieved a large, heavy stone and sat it down outside a large circle she and her friends had made in the sand (to represent their chickadee nest). She announced, "It is for the grown-ups to sit on." Had she not verbalized her intent, the teacher would not have realized that this child had an understanding of size relationships (i.e., large rock– for larger people/adults). In another example, two girls made a large, hexagonal-shaped clock on the ground by connecting several 6' long sticks and adding hands. One girl communicated a fairly sophisticated concept for a young child when she said, "I can tell time with this…by the shadows." When children verbalize their math thinking teachers can assess where they are at developmentally and consider how to best scaffold their learning.

Our findings suggest that children's vocabulary was richer because of the materials they manipulated and/or observed, and the activities they were engaged in. Their vocabulary changed as they observed materials changing (and this flexibility of thinking is important in constructing math knowledge). For example, a child who made a snowball said, "It's a small size, now watch (adding more snow to it), a medium size." A four-year-old who was adding nature items to a container of water to see how high the water would go said, "See, it comes up to here now (and kept adding)…When I add more stuff, the water goes up to here." A child who was watching a friend grind corn with a manual corn grinder observed what happened to the volume as the corn kernels became "dust" and said: "It shreds. Now you need some more. Let's fill it up for you." Children who dug and measured their holes with rulers first noted that they were 12 inches deep, then dug deeper and said the hole was "12 + 5," now "17 inches."

Lesson # 5: The richness of children's math learning in the outdoor classroom was integrally related to the materials available for children to explore and manipulate.

The materials teachers recorded were not part of a prescribed math curriculum. Tables 25–27 in the Findings section illustrate the variety of materials children used. Some of those materials were naturally occurring because they were part of the outdoor environment. Teachers also thoughtfully and intentionally added materials to enhance children's learning experiences (including natural items not readily available in the outdoor classroom, and made items, utensils, and tools). Some materials were living, and children could watch them grow and change through the seasons and over extended periods of time. All of these materials provided tactile, sensory experiences that fully engaged children in the learning process.

The variety of materials and their unique properties provided children with opportunities to make interesting observations, estimations and speculations. Many of the large, long, and heavy materials provided unique learning experiences related to specific math concepts such as weight, length, and height, and provided opportunities for problem-solving and analytical thinking (e.g., how to move them, ways to use them, ways to combine them). Some of the materials (including the availability of materials) changed with the seasons and the unpredictability of the weather and appearance of insects, birds and animals added an additional dimension to children's math exploration and learning. Even the decomposition of natural materials did not discourage children from using them or necessitate discarding them. For example, broken tree cookies or wooden blocks provided opportunities for children to physically work with fractions, make size comparisons and examine a variety of shapes (as well as think about concepts such as "making equal"). Through children's imaginations, fragments of sticks buried in the soil became dinosaur bones that encouraged speculations about size and age (i.e., time concepts), and falling leaves, dying off for the season, became materials to match, collect, count, classify, order by size, and sketch.

Many of the added materials were instrumental to children's math experiences, often related to children's pretend play (e.g., various sizes of containers, buckets, collection bowls, measuring cups, baking tins, binoculars, scales, rulers). Clipboards and writing materials provided opportunities for children to translate three-dimensional observations and experiences to two-dimensional sketches or maps, furthering their understanding of math concepts such as spatial relationships, scale relationships, one-to-one correspondence and geometric shapes. Most of the materials available in the Nature Explore Classroom could be physically manipulated by children, and provided the kinds of hands-on, active experiences that children need to learn math concepts. Since children learn by doing, the tactile and sensory (i.e., hands-on) experiences provided concrete learning that we believe will ultimately help children retain the knowledge they construct and expand that knowledge with future applications.

The results of this study support several authors' beliefs about the importance of providing hands-on materials to support children's learning. For example, Waite-Stupiansky and Stupiansky (1992) emphasized that it is imperative for teachers to "provide concrete math materials that are child–friendly and that can be manipulated in many ways" (p. 8), implying the importance of versatility as well as sensory and tactile learning. The authors asserted that even elementary educators seem to be taking the lead from their early childhood colleagues. They have increasingly recognized the importance of providing hands-on math learning for older children. Basile (1999b) also described the importance of providing young children with concrete, interactive materials to help them understand math concepts. The advantage of the outdoors, she said, is that those materials "can be found everywhere" (p. 157). Richardson (undated) also described the outdoors as a rich environment for learning, because it "includes many materials, both natural and made, that can be used to nurture children's developing math skills." She urged educators to carefully consider selecting materials that will "enrich children's experiences" (p. 26).

Many of the materials children gravitated toward in the outdoor classroom were simple and inexpensive. For example, children sold "statues" (pieces of driftwood) and "pizzas" (tree cookies). They made a clock with sticks; compared the sizes of acorns and icicles; searched for creatures in the worm bin and under logs; used sticks to dig, measure, and draw lines in the dirt; cooked and baked a variety of items in pans and tins; and collected bird feathers, ice chunks and leaves. Those materials provided opportunities for children to develop math skills, including counting, classifying, comparing sizes and shapes, estimating, measuring, and formulating vocabulary. Often educators and parents are swayed by the media's emphasis on purchasing the best, biggest and most expensive educational materials and tools on the market. When it comes to the quality of children's educations, there is subtle pressure to spare no expense. Yet, our data suggest that simple, open-ended materials provided children with hours of rich learning experiences. Diamond and Hopson (1998) suggested that materials do not need to be elaborate to be effective. In fact, they said, "When it comes to providing toys and activities for young children, there is, in general, an inverse relationship between the specificity and elaborateness of a toy and its ability to excite the imagination" (p. 183). While many of the educational materials produced are designed for one particular use, many of the materials children at Dimensions used were open-ended and could be used in a variety of ways.

Brickman and Taylor (1991) cautioned that, "Providing a wide variety of materials, however, is not enough. Children also need time to explore the materials and to use them at their own pace, in ways that are of particular interest to them" (p. 92). Dimensions' teachers understood the importance of providing children with time to explore nature, materials and tools; and to make their plans and change those plans when something else sparked their interest.

The role materials played in supporting children's math learning was a key theme in this study. As critical as materials were to math learning, it is also important to note that having an outdoor space intentionally designed to facilitate young children's learning (physically, cognitively, socially, intrapersonally) was also vital in supporting their math learning. Many of the learning experiences children had were directly related to the design of the space. For example, the Greenhouse provided rich, year-round experiences with seeds, plants, worms and roly polys. The Climbing/Crawling structure and the low brick wall that borders the Messy Materials Area provided children with opportunities to physically experience different levels. The holes intentionally created in the brick wall (that small children could peek or pass materials through) provided opportunities for children to estimate size, experience shapes and learn about negative space. The L-shaped border around the Sand Area (and borders in other areas) provided children with experiences with the concept of perimeter. The pathways through the Nature Explore Classroom gave children opportunities to experience math concepts such as distance, width, length, slope and speed. The brick cave gave children experience with depth and volume. The versatile Open Area gave children opportunities to have timed races. The size of the space, layout and specific landmarks gave children opportunities to create two-dimensional maps based on their three-dimensional observations. The division of the larger outdoor classroom into multiple smaller areas provided a lesson in whole-part relationships and area. Both the Dirt-Digging and Sand Areas gave children opportunities to experience depth, width and volume. The Music Area (including instruments like the akimbira and slap drums) allowed children to learn about rhythm, patterns, and timing. The intentionally designed outdoor classroom space in tandem with interesting and versatile materials provided infinite opportunities for math learning.

Lesson # 6: Our data illustrate that an outdoor classroom environment (and the natural world) provided a powerful context for young children to explore, develop, and hone math skills in ways that could not be replicated in traditional indoor classrooms.

Over the years a number of authors have suggested that the outdoors is a viable venue for teaching math skills, yet little progress has been made in moving math lessons outdoors. For years Basile (1999b) has advocated that context is critical to the learning process, and described the importance of helping young children learn math in context (although she acknowledged that when it comes to math in traditional educational settings, context has not always been an important consideration). She cited studies that have demonstrated the problem with learning without context–researchers have discovered that when students have been presented with math problems, their answers displayed little logical reasoning, precisely because they had minimal context to draw upon. Basile suggested that perhaps it is time for educators to consider teaching math differently and that, "mathematics might be better learned using…(not only different instructional strategies but) a different instructional environment" (p. 156). Basile (1999a) explained the importance of this, and cited others who have advocated for teaching math in context:

> It is crucial that educators create meaningful mathematical environments by providing real-life contexts and setting the foundation for more abstract mathematics. Both the National Council of Teachers of Mathematics and the National Association for the Education of Young Children call for young children to learn in realistic contexts and to

> study the world in which they live. Taking children outdoors gives them real experiences that they might not otherwise have had (p. 12).

Similarly, Whitin, Mills and O'Keefe (1991) suggested that, "Children learn and remember mathematical concepts more easily when they are embedded in meaningful contexts" (p. 23). Stanberry (2014) proposed that, "Combining math with nature (helps children) make more sense of the natural environment around them. It becomes more real. Observing, researching, recording data helps them slow down and focus on the beauty around them. They are shown yet another way that math can be used to describe and interpret our wonderful world" (Stanberry, 2014, p. 3).

Basile (1999a) also discussed the role of observation in children's math learning. Being outdoors, she explained, allows children to "gain knowledge and skills by using their senses to collect information about the world around them." She noted that while educators often view observation as a science skill, making observations is also a critical math skill:

> …direct observation is…an important piece of mathematical learning that is essential for identifying patterns, promoting problem-solving, and developing spatial sense and reasoning. When children become data collectors, they look for patterns and develop reasoning skills that allow them to draw conclusions on the basis of the information that they have not only collected but also observed with their own eyes…as children collect data, they see how prior knowledge about number, size, shape and pattern related to things around them. Data collection builds beliefs about what math is and about what it means to know and do math, and it supports children's view that they are mathematicians. Children who collect data can portray themselves as mathematicians and begin to see how math can be used to solve real problems (pp. 8–9).

In his hallmark text on multiple intelligences, Gardner (1993) concurred with Basile's proposition, and described the outdoors as "a wonderful data source," precisely because "there are items to count, changes to note, patterns to record and observations to log" (p. 76).

Simply stated, children at Dimensions would have had a fraction of the math learning experiences documented in this paper if the indoor classroom was the only space option. While teachers could take some small materials indoors, children's learning experiences would not have been nearly as rich. For example, children observed birds, squirrels, and insects in their natural habitat. They learned about math concepts such as length, diameter, angle, horizontal, vertical, parallel and perpendicular because they had the space to manipulate 6' long poles. The gardens, planted with different sizes, colors and textures of plants offered so many opportunities for counting, classifying, matching, comparing sizes, and learning about whole-part relationships. Various sizes of trees offered opportunities for children to learn about height, diameter, scale relationships, and when they naturally dropped leaves, pinecones and acorns that opened up many more possibilities for hands-on math experiences.

Lesson # 7: Our findings substantiate the importance of child-directed, child-initiated play to fully engage children in purposeful, authentic and meaningful math learning.

There is significant philosophical debate about the merit of a constructivist approach to learning in the early childhood years versus a curriculum-based approach. Advocates of a prescriptive, curriculum-based approach do not believe children can successfully learn math (and other subjects) by simply pursuing their own interests. Our data clearly contradicts this. The observations Dimensions' teacher/co-researchers recorded illustrate rich, authentic and meaningful learning experiences–experiences that were predominantly child-initiated (114/77%), based on children's interests and plans. When classes go outdoors, children have the opportunity to go to the Gathering Area to make their plans. They do this based on their interests and goals that particular day. Even the observations teachers documented that were teacher-initiated were not curriculum-based or formal teaching situations, but suggestions and invitations extended to children, or experiences set up to provide children with specific learning experiences. While concise, perhaps Elkind (1999) said it most eloquently: "Preschoolers have their own curriculum goals" (p. 3). This was evident in our data and in the stories we shared in the Findings section. There is little dispute that the experiences children had in the outdoor classroom were relevant to them, exciting and engaging, and represented the most authentic form of learning.

This study concurs with what numerous authors have shared about the value of child-directed experiences in supporting children's math learning. For example, Clements (2001) suggested that "The most powerful mathematics for a preschooler is usually not acquired while sitting down in a group lesson…but is brought forth…from the child's own self-directed, intrinsically motivated activity" (p. 274). Similarly, Whitin, Mills and O'Keefe (1991) stated that the most "Effective mathematics instruction provides children (with) opportunities to pursue their own interests" (p. 7). They explained how children construct math knowledge based on following their interests and goals:

> Mathematics, like language development, is essentially a process of construction, not acquisition. Learning mathematics involves learning how to use mathematics in one's social world to meet one's particular purposes and intentions…Mathematics is never apart from living. The unfortunate obsession with drill, computation, and rote learning masks the real purposes of mathematics…Mathematics is for recipes to be doubled, plants to be measured, fruit to be divided, prices to be compared, polls to be interpreted, time to be estimated, meals to be ordered, bills to be paid, and marbles to be shared.

> Mathematics is not an act but an event, an experience. Its purposes are not divorced from the lives of children but emanate from the intention of children themselves (p. 6).

Whitin, Mills and O'Keefe also described the importance of capitalizing on children's interests to create "functional" and meaningful math learning:

> By incorporating children's interests…we provide a context in which mathematics is used in an authentic way; these interests connect mathematics to the world of children, emphasizing the functional nature of mathematics. They build on the experience and knowledge that the children have already acquired and enhance children's self-concept by valuing a topic that they already know something about (p. 45).

Waite-Stupiansky and Stupiansky (1992) suggested that, "The best way to develop positive attitudes toward and solid aptitude for math is through…open-ended, child-directed play" (p. 7). Hohmann and Weikart (1995) agreed that math learning happens naturally for young children as they play, explore materials, solve problems, and pursue their interests" and that children learn "spontaneously throughout the day as they follow their own initiatives and interests." They proposed that children learn about numbers by working with manipulatives "rather than by participating in abstract drill–and–practice sessions led by adults" (p. 477).

Gardner (1993) described how young children learn experientially, often of their own accord:

> In the first years of life, young children the world over develop powerful theories and conceptions of how the world works; the physical world and the world of other people. They also develop at least a first–draft of competence with the basic human symbol systems–language, number, music, two-dimensional depiction, and the like. What is striking about these acquisitions is that they do not depend on explicit tutelage. Children

> develop these symbolic skills and these theoretical conceptions largely by dint of their own spontaneous interactions with the world in which they live" (p. 56).

Elkind (1999) reminded educators that early childhood is a critical time for providing a foundation in math, and that it is important to; 1) adapt instruction "to the unique needs, interests, and abilities of young children", 2) "engage children's spontaneous motivation" by tapping into those interests, and 3) to "encourage children's unlimited imagination and curiosity" (p. 6).

Interestingly, many of the activities children were engaged in outdoors were activities that could be included in formal math curriculum (given the availability of materials and space), yet they were not initiated by teachers and were not concepts being formally "taught" via a prescribed curriculum. Children made their plans and decisions, and were fully engaged and joyfully learning as each lesson unfolded.

Lesson # 8: While the majority of children's learning experiences in the outdoor classroom were child-directed, the teacher role was critical in supporting and scaffolding children's math learning.

Previous research tells us that children are innately curious about math and about how the world works, and are interested in applying math principles. Several experts in the early education field have described the teacher's role in supporting children's math learning. For example, Clements (2001) suggested that:

> Preschoolers possess informal mathematical abilities and enjoy using them. Before they enter school, many children develop number and geometry abilities that range from counting objects accurately to finding one's way through the environment, to making shapes. Children use mathematical ideas in everyday life and develop informal mathematical knowledge that is surprisingly complex and sophisticated…This innate

curiosity and even enjoyment creates many teachable moments, and the teacher's role is to nurture children's learning…(Preschoolers) show spontaneous interest in "big" mathematical ideas…they are self-motivated to investigate patterns, shapes, measurement, the meaning of numbers, and how numbers work, but they need assistance to bring these ideas to an explicit level of awareness" (p. 271).

Waite-Stupiansky and Stupiansky (1992) described the teacher's primary role in supporting young children's math learning as building "on children's natural curiosity about shapes, sizes, amounts, and other fundamentals of math (p. 6)," and suggested that the "most important role in fostering math learning (is) to provide the stimulation and encouragement to help children develop and practice their own thinking, to learn math by DOING math, to make mistakes, and in time, to learn from them." (p. 9). The High/Scope Foundation (2003) proposed that in order to support early math learning, teachers need to provide children with "the words, materials, and experiences appropriate for math learning at their age so they can investigate size, quantity, categorization, patterns, space, speed and sequence on their own terms." Brinkman and Taylor (1991) described the teacher's role as encouraging "children to think about what they are doing and to put what they know into their own words" (p. 92).

Our findings illustrate that skilled, caring and committed teachers at Dimensions provided the kind of nurturing support these authors described. Tables 28–31 identify the many ways teachers fostered children's interest in math, challenged them to think about math principles, and encouraged them to share their math knowledge as it evolved. At appropriate times, teachers provided information that scaffolded children's math learning (e.g., about weight, shapes, numbers, quantity, how to keep score, measurement). They provided vocabulary when children were stuck or learning new concepts. They provided materials that supported specific math learning (e.g., beans to sort, count and classify; seed packets to match to plants; photos and labels to match to plants; binoculars to observe birds in a distance–and count them and record their findings; varieties of lettuces to compare by their attributes; containers to dump and fill) and provided tools to help children experience math concepts (e.g., shovels for digging, long poles to manipulate and extend reach, rulers for measuring, paper and writing utensils to represent their experiences two-dimensionally). Teachers provided physical assistance when needed to allow children to successfully carry out their plans. They served as resource individuals, helping children learn where they could find information that would extend their math knowledge (i.e., reference books). They extended invitations and challenges to children that provided opportunities for them to apply math principles (e.g., to write numbers sequentially on stumps with chalk to show directionality for an obstacle course, to find matching insects, to survey peers and tally responses, to create maps and sketch their work, to break a record finding and counting roly polys). And often, at children's invitations, they participated in their play. It was teacher's excitement, interest, and engagement that encouraged children to share their discoveries, whether through their stories, their speculations, and/or their sketching and artwork. It was teacher's interest, questions, and documentation that validated what children were doing and saying, allowing children to develop confidence as "experts."

Copley (2000) suggested that, "The teacher's role in fostering children's reasoning processes is critical," and explained that teachers should ask children "questions that require investigation and reasoning." Some of those questions might include:

Are you sure?

How do you know?

Why do you think…?

What else can you find that works like this?

What would happen if…?

I wonder how this could be changed?

What would the pattern be?

What if…?

I wonder why…?

Perhaps it's because…? (p. 37).

Basile (1999b) suggested that the questions teachers ask should be designed to "help children think about the important mathematical understanding involved in data collection as well as clarify their own perceptions of the natural world"…and "the best questions are open-ended questions that begin with "how" or "what," or ask children, "what do you think?" (p. 11).

When we examined teachers' Nature Notes, we identified the kinds of questions teachers asked children to help facilitate their math learning. We provided a sampling of those questions in Table # 28. As that table illustrates, teachers asked children simple how, what and why questions. They used math vocabulary in their questions (e.g., words like heavy, long, big, feet, tall, triangle, shape) that children could hear (and learn from in the context of their activities). Teachers did not necessarily expect children to reply with precise or "correct" answers, but asked questions to intentionally stimulate math thinking. For example, teachers asked, "How big…," "How long…," "How heavy…," "How many…." They asked questions like, "What did you do first?" to help children think about sequencing, "What shape is it?" and "What color is it?" One teacher asked a child, "Can you take a scientific guess?" (implying there was no wrong answer and encouraging speculation). Sometimes teachers asked "I wonder…" questions, encouraging children to wonder with them. Teachers needed to be fully engaged and present with children, and listening intently to their play, to know when to ask relevant questions.

It was significant that teachers did not always expect precise or correct responses to questions or correct children's use of math vocabulary. Teachers were well aware that children's ability to reason mathematically develops over time. Several authors have discussed the importance of allowing children to speculate and guess as they construct math knowledge through active learning experiences. For example, Waite-Stupiansky and Stupiansky (1992) said it is important that teachers accept children's math reasoning, "even when it may seem 'wrong' or illogical" because that acceptance "will give them the confidence to keep thinking, questioning and sharing" (p. 6). They suggested that as teachers observe young children's math–related explorations and listen to their stories, the message teachers should communicate through their questions and responses "should be, 'Tell me what you think,' and not, 'give me the right answer'" (p. 8). Teachers need to "encourage children to talk about number concepts without fear of being wrong." They will "make many incorrect statements in the process of learning math, just as they do when learning to talk. Correcting them may discourage children from offering their own solutions. Often when children are using an incorrect term, they conceptually have the right idea" (p. 9). When teachers ask questions with many possible responses–the kind that ask 'how…' and 'why…' and 'what do you think'–their communication tells children that they "are not looking for the 'right answer', (and) when children know that, they feel freer to explore and to use materials in the ways they're capable of for their age–and to share their thinking" (p. 17).

Hohmann and Weikart (1995) posited that, "as professional educators, adults are responsible for establishing a psychologically safe and hence logical environment for children. At the same time…they also understand the importance of valuing and accepting children's intuitive explanations, even if these explanations are inaccurate by adult standards" (p. 448). Similarly, the High/Scope Foundation (1993) suggested that teachers need to "recognize the importance of valuing and accepting children's explanations for their findings, even if some don't quite fit within adult standards" (p. 5).

It is also important for early childhood teachers to feel competent and confident in supporting children's math learning. Today's teachers may be some of the very students who grew up fearing math or experienced math anxiety in school. Because of their lack of confidence in "doing math" and teaching math concepts, they may shy away from engaging in math-related discussions and activities with children. Waite-Stupiansky and Stupiansky (1992) suggested that, "what young math thinkers need most is an adult who fosters their interest in math." Yet many teachers are hesitant or even intimidated at the thought of teaching math, because for many "the word 'math' triggers feelings of inadequacy or 'math anxiety' – a general feeling of not being comfortable with or confident in math" (p. 7). The authors remind teachers that they do not need to know all the answers, and in fact, it may be better if they do not, so teachers and children "can enjoy the process of learning and discovering math together" (p. 7).

We did not necessarily hear teachers express fears about math in the focus group interviews we conducted, however we did hear teachers describe an overall lack of awareness (potentially related to fear and or lack of confidence?) about ways the outdoor classroom environment and materials supported young children's math learning. Math skills were rarely on their radar when they documented children's learning. It was not until we dedicated time to analyzing teachers' Nature Notes (that they brought with them to the focus group interviews) and discussed those with them that many teachers began to identify the math skills children were developing in the outdoor classroom. It was a few months after the focus group interviews that one teacher confessed she had not thought much about documenting math skills, but added, "Now I watch children and I see math everywhere!" This has implications for teacher training if early education programs are committed to supporting children's math learning.

Recommendations

Based on the findings of this study, we offer several practical recommendations for key audiences that are interested in supporting young children's math learning.

• Administrators and board members associated with early education programs should discuss the value-added learning benefits of intentionally designed outdoor spaces, and evaluate the feasibility of creating outdoor learning spaces for children. Their discussions should involve parents and the community and it may benefit them to consult with organizations like Nature Explore and/or landscape architects and educators who assist with outdoor classroom concept designs. If creating outdoor spaces is feasible, we encourage sites to pursue funding and develop strategic plans for constructing those spaces.

• Considering what is best for children, we encourage teachers to commit to taking children outdoors daily and giving them time to play in nature and investigate materials in creative and unconventional ways.

• Teachers and administrators who work at schools that have outdoor spaces should discuss the kinds of learning experiences and skill development they want to foster (in math and other domains) and develop a list of materials that will support that learning in active, hands-on, meaningful ways (then explore the availability of those materials and how to provide them for their outdoor spaces).

• Early education programs should consider offering teacher training sessions that specifically address ways to recognize and support children's math learning in outdoor settings. As part of this, it would be interesting to have teachers document children's activities and interactions outdoors, then analyze their documentation to specifically examine ways children are constructing math knowledge.

• Teachers need to be intentional about the kinds of math vocabulary they use in their interactions with children and when it is appropriate, scaffold children's learning by providing vocabulary words for the math concepts children are learning.

• Teachers need to allow children the time to engage in spontaneous, open-ended, self-directed play–to initiate their plans, select materials that support their play, even change their plans as their interests change. Teachers should carefully observe children as they pursue their interests, be ready to participate in children's activities (at their invitation), and look for teachable moments when it is appropriate to scaffold children's math learning.

• Early education programs with outdoor classrooms should consider developing a system to formally document children's activities and interactions as they play and work outdoors, and a system to examine the data they collect. This will be useful for administrators who advocate for the value of taking children outdoors (and ask for funding); it will help teachers tangibly see children's learning and where they are at developmentally; it will help teachers better understand their role in scaffolding children's learning, and help them more specifically communicate the skills children are learning to parents.

• Faculty in teacher education programs might develop curriculum to prepare future teachers to teach in outdoor environments; in particular to help new teachers become comfortable with the outdoors as a classroom, and to become comfortable supporting children's skill development in subject areas such as math.

Based on our findings, we also offer the following recommendations for future research:

• Researchers might conduct a longitudinal study that follows children who have attended Dimensions Early Education Programs into primary school, to examine how well those children do on math assessments compared to children who did not have outdoor experiences in preschool;

• Researchers might interview parents who had children at Dimensions, to explore their perceptions of their children's development in math, in relation to their preschool preparation;

• Researchers might compare preschool children's math preparation in programs that do not have outdoor spaces with programs that have constructed outdoor classrooms;

• Researchers might compare math learning in preschool programs that are curriculum-based with programs that do not use specific math curriculum (and are more open-ended and child-directed);

• Researchers might examine preschool teachers' attitudes about children's math learning outdoors, and teachers' perceived comfort and skill level to specifically support children's math development; and

• Researchers might replicate this study with several similar preschool programs that have Nature Explore Classrooms to determine whether these findings are consistent with a larger sample.

Epilogue

"One day I woke up and had a math brain! All I ever wanted to do was math every and all day. It was so fun. Do you love math too? I like it so much. Do you know 2 + 2 or 4 + 4? I used my brain to be an engineer."

~ Owen Limpach, 7 years old,

First Grade Book: One Day…(May 2014)

References

Baloglu, M., & Kocak, R. (2006). A multivariate investigation of the differences in mathematics Anxiety. *Personality and Individual Differences*, 40, pp. 1325-1335.

Basile, C.G. (1999a). Collecting data outdoors: Making connections to the real world. *Teaching Children Mathematics*, 6(1), pp. 8-12.

Basile, C.G. (1999b). The outdoors as a context for mathematics in the early years. In *Mathematics in the early years*, by Juanita V. Copley (Ed.). Washington, D.C.: NAEYC.

Bezanson, C. & Killion, J. (Spring 2001). Moving math outdoors. Green Teacher, 64, pp. 31-33.

Bergen, D. (Spring 2002). The role of pretend play on children's cognitive development. *Early Childhood Research and Practice*, 4(1).

Birgin, O., Baloglu, M., Catlioglu, H., & Gurbuz, R. (2010). An investigation of mathematics Anxiety among sixth through eighth grade students in Turkey. *Learning and Individual Differences*, 20, pp 654-658.

Bohling, V., Saarela, C., & Miller, D. L. (2010). *This never would have happened indoors: Supporting preschool-age children's learning in a Nature Explore Classroom in Minnesota.* Forest Lake, MN. (unpublished paper at www.dimensionsfoundation.org).

Bohling, V., Saarela, C., & Miller, D. L. (2012). *Supporting parent engagement in children's learning outdoors: A single case study.* Lincoln, NE: The Arbor Day Foundation.

Brickman, N.A. & Taylor, L.S. (Eds.) (1991). *Supporting young learners: Ideas for preschool and daycare providers.* High/Scope Educational Research Foundation. Ypsilanti, MI: High/Scope Press.

Broda, H.W. (2007). *Schoolyard-enhanced learning: Using the outdoors as an instructional*

Tool, K-8. Portland, ME: Stenhouse.

Brown, S. & Vaughn, C. (2009). *Play: How it shapes the brain, opens the imagination, and invigorates the soul*. NY: Avery.

Capraro, M.M., Capraro, R.M., & Henson, R.K. (2001). Measurement error of scores on mathematics anxiety rating scales across studies. *Educational and Psychological Measurement*, 61, pp. 373-386.

Clements, D.H. (2001). Mathematics in preschool. In *Teaching children mathematics*, The National Council of Teachers of Mathematics, www.nctm.org.

Clements, D.H. (October, 2013). Math in the early years: A strong predictor for later school success. Education Commission of the States. *The Progress of Education Reform*, 14(5).

Clements, D.H., Baroody, A.J. & Sarama, J. (2013) Background research on early mathematics. National Governor's Association Center Project on Early Mathematics. Retrieved May 15, 2014 @ http://www.nga.org/files/live/sites/NGA/files/pdf/2013/1311SEME-Background.pdf.

Cobb, E. (1977). *The ecology of imagination in childhood*. NY: Columbia University Press.

Copley, J.V. (2000) The young child and mathematics. Washington, D.C.: NAEYC.

Crain, W. (2001). How nature helps children develop. *Montessori Life*, Summer 2001.

Creswell, J.W. (2013). *Qualitative inquiry and research design: Choosing among five Approaches* (3rd Ed.). Thousand Oaks, CA: Sage Publications.

Diamond, M. & Hopson, J. (1998). *Magic trees of the mind: How to nurture your child's intelligence, creativity, and healthy emotions from birth through adolescence*. NY: Plume/Penguin Putnam.

Dillon, J., Morris, M., O-Donnell, L., Reid, A., Rickinson, M., & Scott, W. (2005). Engaging and learning with the outdoors–The final report of the Outdoor Classroom in a Rural Context action research project, *National Foundation for Education Research*, April, 2005.

Elkind, D. (1999). Dialogue on early childhood science, mathematics, and technology education: A context for learning. *The American Association for the Advancement of Science*. Retrieved 10/16/08 at http://www.project2061.org/publications/earlychild/online/ context/elkind.htm.

Elkind, D. (2007). *The power of play: How spontaneous, imaginative activities lead to happier, healthier children*. Cambridge, MA: DaCapo Press.

Gardner, H. (1993). *Frames of mind: The theory of multiple intelligences.* NY: Basic Books.

Gellens, S. (2007). *Activities that build the young child's brain*. Early Childhood Association of Florida, Inc.

Ginsburg, K. (2007). The importance of play in promoting healthy child development and maintaining strong parent-child bonds. In American Academy of Pediatrics, 119(1), pp. 182-191.

Googal Learning (undated). *Using math in nature*. Retrieved 10/22/2008 at http://www. Googalpower.com/context/ecomath/math-in-nature.

High/Scope (no author), 2003). Key experiences: Classification, seriation, and number. High/Scope Educational Research Foundation. Ypsilanti, MI: High/Scope Press.

Hirsh-Pasek, K. & Golinkoff, R.M. (2003). *Einstein never used flashcards*. Rodale.

Hirsh-Pasek, K., Golinkoff, R.M. Berk, L.E. & Singer, D.G. (2009). *A mandate for playful learning in preschool: Presenting the evidence*. NY: Oxford University Press.

Hohmann, M. & Weikart, D.P. (1995). *Educating young children: Active learning practices* *for preschool and child care programs*. High/Scope Educational Research Foundation. Ypsilanti, MI: High/Scope Press.

Keeler, R. (2008). *Natural playscapes*. Redmond, WA: Exchange Press.

Lincoln, Y.A. & Guba, E.G. (1985). Naturalistic inquiry. Newbury Park, CA: Sage.

Linn, S. (2008). *The case for make-believe: Saving play in a commercialized world*. NY: The New Press.

Louv, R. (January, 2009). No more "Nature-Deficit Disorder". Psychology Today. Retrieved May 16, 2014 at http://www.psychologytoday.com/blog/people-in-nature/200901/no-more-nature-deficit-disorder.

Merriam, S.B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco, CA: Jossey-Bass.

Miller, D.L. (2007). *The seeds of learning: Young children develop important skills through their gardening activities at one Midwestern Early Education Program. Applied Environmental Education and Communication*, 6, 49-66.

Miller, D.L., Tichota, K., & White, J. (2013). *Young children's authentic play in a Nature Explore classroom supports foundational learning: A single case study*. Lincoln, NE: Dimensions Educational Research Foundation.

Moore, R. & Wong, H. (1997). *Natural learning: Rediscovering nature's way of teaching*. Berkeley, CA: MIG Communications.

Newstead, K. (1998). Aspects of children's mathematics anxiety. *Educational Studies in Mathematics*, 36, pp. 53-71.

Ostrow, N. (August 2008). *Preschool tied to higher math skills*. The Boston Globe. Retrieved 10/16/2008 at http://www.boston.com/news/education/K_12/articles/2008/08/29/

preschool_tied_to_higher_math_skills.

Paley, V.G. (2004). *A child's work: The importance of fantasy play*. IL: The University of Chicago Press.

Palmer, J. (1993). Development of concern for the environment and formative experiences of educators. *Journal of Environmental Education*, 24, 26-30.

Pappas, T. (1989). *The joy of mathematics: Discovering mathematics all around you*. San Carlos, CA: Wide World Publishing/Tetra.

Peterson, P.E., Woessmann, L., Hanushek, E.A., & Lastra-Anadon, C.X. (August 2011). Globally challenged: Are U.S. students really ready to compete? Harvard's Program on Education Policy and Governance & Education Next, Harvard Kennedy School.

Porter, N., (August 2012). *High turnover among early childhood educators in the United States*. Child Research Net. Retrieved April 3, 2014 from www.childresearch.net/projects/ecec/2012_04.html.

Pyle, R. (2002). Eden in a vacant lot: Special places, species, and kids in community of life. In *Children and nature: Psychological, sociological and evolutionary investigations*, Kahn, P.H. & Kellert, S.R. (Eds.). Cambridge, MA: MIT Press.

Raytheon (2012). Math Relevance to U.S. Middle School Students: A Survey Commissioned by Raytheon Company. Conducted by Research Now.

Richardson, G.R. (undated). Open up to outdoor mathematics. Learning Through Landscapes. Retrieved May 10, 2014 at https://www.hope.ac.uk/media/liverpoolhope/contentassets/documents/education/media,25538,en.pdf

Rosenow, N., Wike, J., & Cuppens, V. (2007). *Learning with nature idea book: Creating nurturing outdoor spaces for children.* Lincoln, NE: Arbor Day Foundation and Dimensions Educational Research Foundation.

Rosenow, N., Reeble, T., & Kiewra, C. (2011). *Growing with nature: Supporting whole-child learning in outdoor classrooms.* Lincoln, NE: Arbor Day Foundation and Dimensions Educational Research Foundation.

Sobel, D. (1998). *Mapmaking with children: Sense of place education for the elementary years*. Portsmouth, NH: Heinemann.

Stake, R.E. (1995). *The art of case study research*. Thousand Oaks, CA: Sage.

Stake, R.E. (2007). *Reconsidering generalization and theory in case study research*. Paper Presentation at the annual meeting of the American Educational Research Association, Chicago, IL.

Stanberry, K. (2014). *Early math matters: Preparing preschoolers to succeed*. Retrieved 5/13/14 At http://www.getreadytoread.org/early-learning-childhood-basics/early-math/

Suinn, R.M., Taylor, S., & Edwards, R.W. (1998). Suinn mathematics anxiety rating scale for elementary school students (MARS-E): Psychometric and normative data. *Educational and Psychological Measurement*, 48, pp. 979-986.

Taylor, A.F., Kuo, F.E., & Sullivan, W.C. (2001). Coping with ADD: The surprising connection to green play settings. *Environment and Behavior*, 33(1), 54-77.

Veselack, E., Cain-Chang, L., & Miller, D. L. (2011). *Young children develop foundational skills through child-initiated experiences in a Nature Explore Classroom: A single case study in La Canada, CA.* (unpublished study at www.dimensionsfoundation.org).

Vukovic, R.K., Kieffer, M.J., Bailey, S.P. & Harari, R.R. (2012). Mathematics anxiety in young children: Concurrent and longitudinal associations with mathematical performance. *Contemporary Educational Psychology*, 38, pp. 1-10.

Waite, S. (Ed.), (2011). *Children learning outside the classroom: From birth to eleven*. Thousand Oaks, CA: Sage.

Waite-Stupiansky, S. & Stupiansky, N.G. (1992). *Learning through play-Math: A practical guide for teaching young children*. NY: Scholastic Incorporated.

Wells, N.M. & Evans, G.W. (2003). Nearby nature: A buffer of life stress among rural children. *Environment and Behavior*, 35(3), 311-330.

Whitin, D.J.; Mills, H. & O'Keefe, T. (1991). *Living and learning mathematics: Stories and strategies for supporting mathematical literacy*. Portsmouth, NH: Heinemann.

Wilson, R.A. (1997). The wonders of nature: Honoring children's ways of knowing. *Early Childhood News*, 6(19).

Yin, R. K., (2014). *Case study research: Design and methods* (5th Ed.). Thousand Oaks, CA: Sage.

Appendix 1

DIMENSIONS®
Educational Research Foundation
dimensionsfoundation.org

Documentation Form

☐ **Nature Notes**
☐ **Visual Notes**
☐ **Other Developmental Notes**

Teacher/Observer: ____________________

Date: ______/ ______/______ Time of Day: ____________________

Brief description of activity :

☐ Teacher Initiated Activity
☐ Child Initiated Activity

How Long Observed: ____________________

Why you believe this is significant:

Weather Conditions:

____

Child(ren)	Age/Birthdate	Gender

Location of Activity

☐ Block area
☐ Toy area
☐ House area
☐ Art area
☐ Materials Exploration
☐ Science area
☐ Snack
☐ Climbing/Crawling area
☐ Messy Materials area
☐ Block/Building area
☐ Nature Art area
☐ Garden/Pathways
☐ Greenhouse
☐ Music/Movement area
☐ Open area
☐ Gathering area
☐ Dirt Digging area
☐ Sand area
☐ Brick Wall

Resources/Materials

☐ Akambira
☐ Nature Art Table
☐ Tree Cookies
☐ Tree Blocks
☐ Square/Rectangle Blocks
☐ Rainstick
☐ Scarves
☐ Garden Tools
☐ Clipboards
☐ Magnifying Glasses
☐ Tape Measures
☐ Water
☐ Shovel
☐ Rake
☐ Hand Trowel
☐ Nature Image Cards
☐ Writing Materials
☐ Unit Blocks
☐ Large Hollow Blocks
☐ Fabric/Clips
☐ Paint/Watercolors
☐ Reusables
☐ Other____________
☐ Other____________

Key skills children are developing through using Dimensions' resources

Skills	Other/Comments
Visual-Spatial ☐ observing closely ☐ noticing patterns, details, textures, colors, shapes, sizes ☐ discriminating between objects/types/sizes ☐ developing figure-ground perspective ☐ seeing from multiple perspectives ☐ reading and following maps and recognizing landmarks ☐ understanding concepts such as inside, outside, over, under, around, through ☐ making visual analogies (X looks like Y) ☐ learning about dimensionality ☐ creating maps (spatial orientation) ☐ seeing, storing, retrieving visual images ☐ identifying similarities and differences	**Other/Comments**
Language/Literacy ☐ recognizing and using symbols ☐ telling stories ☐ reading ☐ conversing with other children and adults ☐ recognizing that print has meaning/is verbal language translated to written form ☐ practicing letters, forming words, formatting them on the page ☐ naming ☐ spelling ☐ creating pledges, poems, songs	**Other/Comments**
Science ☐ understanding seasons/lifecycles ☐ learning about plant life, pond life, insects, birds, animals, habitats ☐ learning about hibernation, migration, metamorphosis ☐ formulating research questions/hypotheses ☐ conducting experiments ☐ learning about cause-and-effect relationships	**Other/Comments**
Mathematics ☐ counting ☐ matching ☐ learning geometric shapes ☐ understanding whole-part relationships ☐ understanding scale relationships ☐ understanding diameter ☐ experiencing area and volume ☐ classifying ☐ seriating ☐ understanding time concepts ☐ sequencing ☐ estimating distance ☐ recognizing symmetry ☐ understanding perimeter	**Other/Comments**
Kinesthetic/Body Awareness ☐ using body as a tool and learning to use tools ☐ developing fine and gross motor skills (small and large muscle movement) ☐ developing muscle memory/concepts cemented with repeat experiences ☐ turning body into shapes (helps internalize learning) ☐ creating dances (creative and emotional expression) ☐ experiencing textures and shapes of natural materials (sensory/touch) ☐ developing balance and knowledge of stability ☐ navigating through space (awareness of body in space and proximity of body to objects)	**Other/Comments**
Social/Interpersonal ☐ learning cooperation and teamwork ☐ resolving conflicts ☐ communicating desires, needs, ideas to others ☐ learning to share, negotiate ☐ interacting/collaborating with adults ☐ sharing knowledge and expertise with others (children, teachers, parents)	**Other/Comments**
Intrapersonal ☐ developing critical thinking, questioning skills, abstract thinking ☐ developing respect/reverence for the environment ☐ developing a sense of ownership and responsibility to become good stewards of the environment ☐ developing self confidence, pride, self-efficacy ☐ taking initiative ☐ expressing emotion ☐ solving problems ☐ expressing creativity ☐ taking appropriate risks ☐ conquering fears ☐ making decisions	**Other/Comments**
Construction/Engineering ☐ stacking ☐ making balance ☐ bridging ☐ ramping ☐ making tunnels ☐ making lines (straight, curved, zig-zag, etc.) ☐ making walls ☐ cornering ☐ emptying and filling ☐ supporting ☐ making an opening ☐ making symmetrical ☐ propping ☐ stepping ☐ making enclosures ☐ covering	**Other/Comments**
Creative Representation ☐ making representational models (3D) ☐ making representational drawings, sketches, paintings (2D) ☐ pretending/role playing ☐ using natural objects to represent other things (transference)	**Other/Comments**
Music ☐ keeping a beat ☐ creating music ☐ moving to music ☐ singing songs ☐ matching a pitch	**Other/Comments**